

# CHAPTER 3

## Exponential and Logarithmic Functions

### Section 3.1 Exponential Functions and Their Graphs

- You should know that a function of the form  $y = a^x$ , where  $a > 0$ ,  $a \neq 1$ , is called an exponential function with base  $a$ .
- You should be able to graph exponential functions.
- You should be familiar with the number  $e$  and the natural exponential function  $f(x) = e^x$ .
- You should know formulas for compound interest.
  - (a) For  $n$  compoundings per year:  $A = P\left(1 + \frac{r}{n}\right)^{nt}$ .
  - (b) For continuous compoundings:  $A = Pe^{rt}$ .

#### Vocabulary Check

1. algebraic
2. transcendental
3. natural exponential, natural
4.  $A = P\left(1 + \frac{r}{n}\right)^{nt}$
5.  $A = Pe^{rt}$

1.  $(3.4)^{6.8} \approx 4112.033$       2.  $1.2^{1/3} \approx 1.063$       3.  $5^{-\pi} \approx 0.006$

4.  $8.6^{-3(-\sqrt{2})} = 8.6^{3\sqrt{2}} \approx 9220.217$

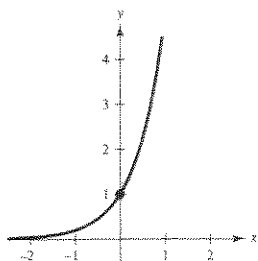
5.  $g(x) = 5^x$

$x$	-2	-1	0	1	2
$y$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25

Asymptote:  $y = 0$

Intercept:  $(0, 1)$

Increasing



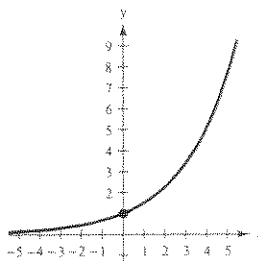
6.  $f(x) = \left(\frac{3}{2}\right)^x$

$x$	-2	-1	0	1	2
$y$	$\frac{4}{9}$	$\frac{2}{3}$	1	$\frac{3}{2}$	$\frac{9}{4}$

Asymptote:  $y = 0$

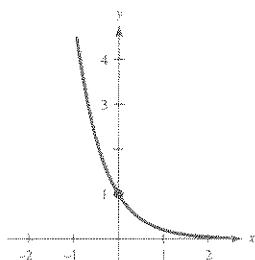
Intercept:  $(0, 1)$

Increasing



7.  $f(x) = \left(\frac{1}{5}\right)^x = 5^{-x}$

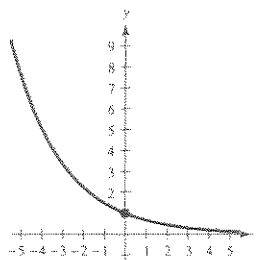
$x$	-2	-1	0	1	2
$y$	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$



Asymptote:  $y = 0$   
 Intercept:  $(0, 1)$   
 Decreasing

8.  $h(x) = \left(\frac{3}{2}\right)^{-x}$

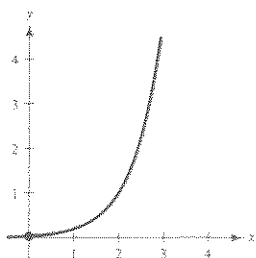
$x$	-2	-1	0	1	2
$y$	$\frac{9}{4}$	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$



Asymptote:  $y = 0$   
 Intercept:  $(0, 1)$   
 Decreasing

9.  $h(x) = 5^{x-2}$

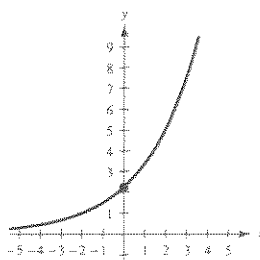
$x$	-1	0	1	2	3
$y$	$\frac{1}{125}$	$\frac{1}{25}$	$\frac{1}{5}$	1	5



Asymptote:  $y = 0$   
 Intercept:  $(0, \frac{1}{25})$   
 Increasing

10.  $g(x) = \left(\frac{3}{2}\right)^{x+2}$

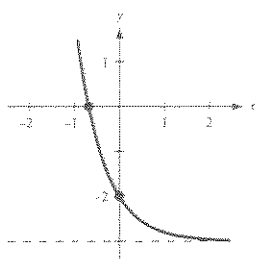
$x$	-4	-3	-2	-1	0
$y$	$\frac{4}{9}$	$\frac{2}{3}$	1	$\frac{3}{2}$	$\frac{9}{4}$



Asymptote:  $y = 0$   
 Intercept:  $(0, \frac{9}{4})$   
 Increasing

11.  $g(x) = 5^{-x} - 3$

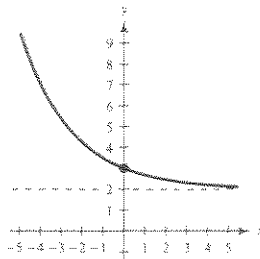
$x$	-1	0	1	2
$y$	2	-2	$-2\frac{4}{5}$	$-2\frac{24}{25}$



Asymptote:  $y = -3$   
 Intercepts:  
 $(0, -2)$ ,  $(-0.683, 0)$   
 Decreasing

12.  $f(x) = \left(\frac{3}{2}\right)^{-x} + 2$

$x$	-2	-1	0	1	2
$y$	$\frac{17}{4}$	$\frac{7}{2}$	3	$\frac{8}{3}$	$\frac{22}{9}$



Asymptote:  $y = 2$   
 Intercept:  $(0, 3)$   
 Decreasing

13.  $f(x) = 2^{x-2}$  rises to the right.

Asymptote:  $y = 0$

Intercept:  $(0, \frac{1}{4})$

Matches graph (d).

14.  $f(x) = 2^{-x}$  is positive and decreasing.

Matches graph (a).

15.  $f(x) = 2^x - 4$  rises to the right.

 Asymptote:  $y = -4$ 

 Intercept:  $(0, -3)$ 

Matches graph (c).

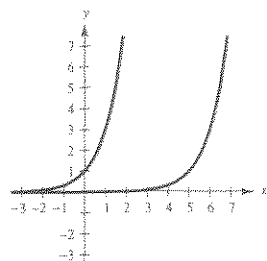
16.  $f(x) = 2^x + 1$  is increasing and has  $(0, 2)$  intercept.

Matches graph (b).

17.  $f(x) = 3^x$

$$g(x) = 3^{x-5} = f(x-5)$$

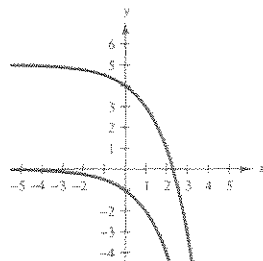
Horizontal shift five units to the right



18.  $f(x) = -2^x$

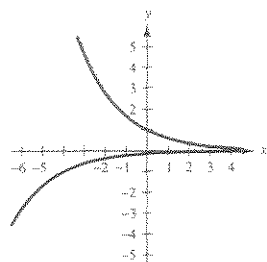
$$g(x) = 5 - 2^x = 5 + f(x)$$

Vertical shift five units upward



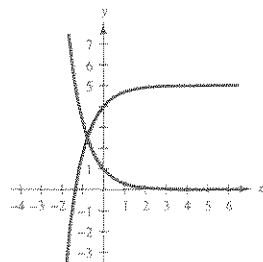
19.  $f(x) = \left(\frac{3}{5}\right)^x$

$$g(x) = -\left(\frac{3}{5}\right)^{x+4} = -f(x+4)$$

 Horizontal shift four units to the left, followed by reflection in  $x$ -axis


20.  $f(x) = 0.3^x$

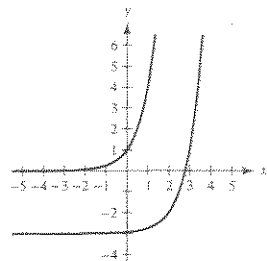
$$g(x) = -0.3^x + 5 = -f(x) + 5$$

 Reflection in  $x$ -axis followed by vertical shift five units upward


21.  $f(x) = 4^x$

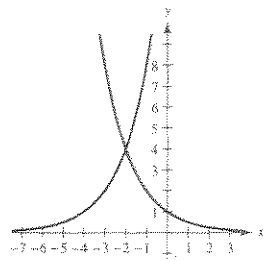
$$g(x) = 4^{x-2} - 3 = f(x-2) - 3$$

Horizontal shift two units to the right followed by vertical shift three units downward



22.  $f(x) = \left(\frac{1}{2}\right)^x$

$$g(x) = \left(\frac{1}{2}\right)^{-(x+4)}$$

 Reflection in the  $y$ -axis followed by left shift of four units


23.  $e^{9.2} \approx 9897.129$

24.  $e^{-(-3/4)} = e^{3/4}$

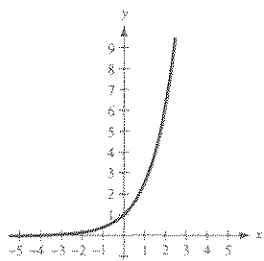
$$\approx 2.117$$

25.  $50e^{4(0.02)} \approx 54.164$

26.  $-5.5e^{-200} = 7.611 \times 10^{-87} \approx 0$

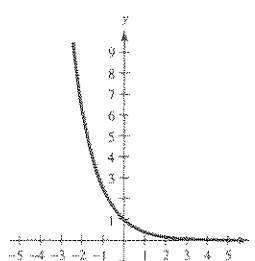
27.  $f(x) = \left(\frac{5}{2}\right)^x$

$x$	-2	-1	0	1	2
$f(x)$	0.16	0.4	1	2.5	6.25

Asymptote:  $y = 0$ 

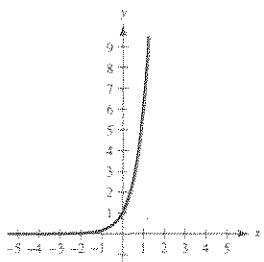
28.  $f(x) = \left(\frac{5}{2}\right)^{-x}$

$x$	-2	-1	0	1	2
$f(x)$	6.25	2.5	1	0.4	0.16

Asymptote:  $y = 0$ 

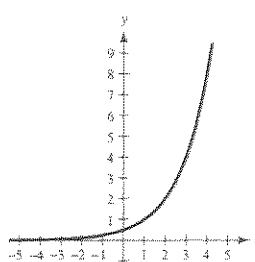
29.  $f(x) = 6^x$

$x$	-2	-1	0	1	2
$f(x)$	0.03	0.17	1	6	36

Asymptote:  $y = 0$ 

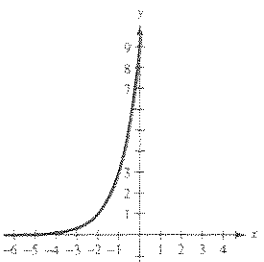
30.  $f(x) = 2^{x-1}$

$x$	-1	0	1	2	3
$f(x)$	0.25	0.5	1	2	4

Asymptote:  $y = 0$ 

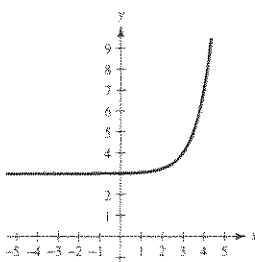
31.  $f(x) = 3^{x+2}$

$x$	-3	-2	-1	0	1
$f(x)$	0.33	1	3	9	27

Asymptote:  $y = 0$ 

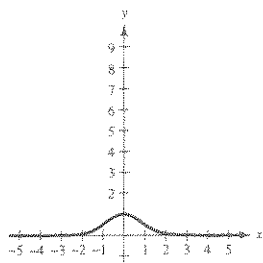
32.  $f(x) = 4^{x-3} + 3$

$x$	0	1	2	3	4	5
$f(x)$	3.016	3.063	3.25	4	7	19

Asymptote:  $y = 3$

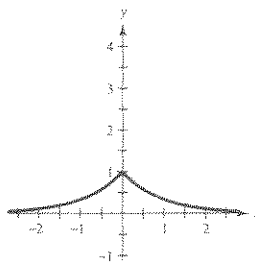
33.  $y = 2^{-x^2}$

$x$	-2	-1	0	1	2
$y$	0.06	0.5	1	0.5	0.06


 Asymptote:  $y = 0$ 

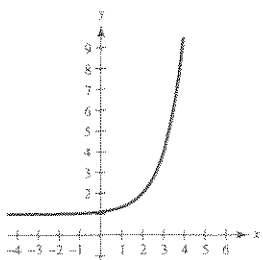
34.  $y = 3^{-|x|}$

$x$	-2	-1	0	1	2
$y$	0.11	0.33	1	0.33	0.11


 Asymptote:  $y = 0$ 

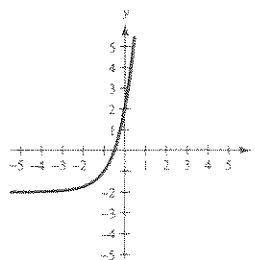
35.  $y = 3^{x-2} + 1$

$x$	-1	0	1	2	3	4
$y$	1.04	1.11	1.33	2	4	10


 Asymptote:  $y = 1$ 

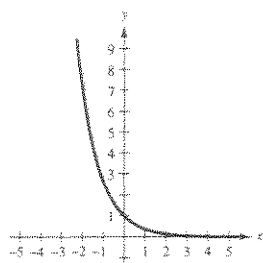
36.  $y = 4^{x+1} - 2$

$x$	-2	-1	0	1	2
$y$	-1.75	-1	2	14	62


 Asymptote:  $y = -2$ 

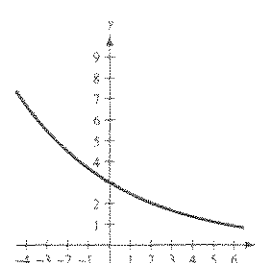
37.  $f(x) = e^{-x}$

$x$	-2	-1	0	1	2
$f(x)$	7.39	2.72	1	0.37	0.14


 Asymptote:  $y = 0$ 

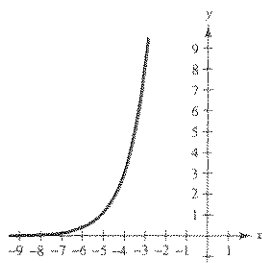
38.  $s(t) = 3e^{-0.2t}$

$t$	-1	0	1	2	3	4
$s(t)$	3.66	3	2.46	2.011	1.65	1.35


 Asymptote:  $y = 0$

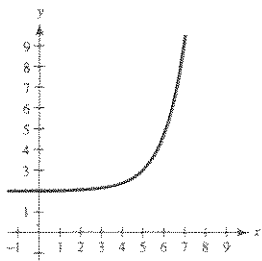
39.  $f(x) = 3e^{x+4}$

$x$	-6	-5	-4	-3	-2
$f(x)$	0.41	1.10	3	8.15	22.17

Asymptote:  $y = 0$ 

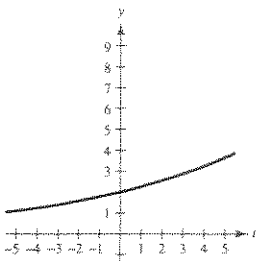
41.  $f(x) = 2 + e^{x-5}$

$x$	3	4	5	6	7
$f(x)$	2.14	2.37	3	4.72	9.39

Asymptote:  $y = 2$ 

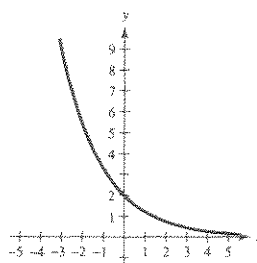
43.  $s(t) = 2e^{0.12t}$

$t$	-2	-1	0	1	2
$s(t)$	1.57	1.77	2	2.26	2.54

Asymptote:  $y = 0$ 

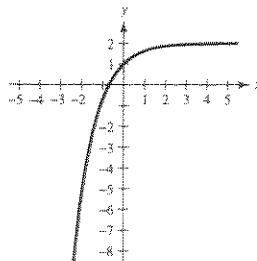
40.  $f(x) = 2e^{-0.5x}$

$x$	-2	-1	0	1	2
$f(x)$	5.44	3.30	2	1.21	0.74

Asymptote:  $y = 0$ 

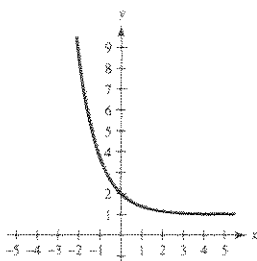
42.  $g(x) = 2 - e^{-x}$

$x$	-2	-1	0	1	2
$g(x)$	-5.39	-0.72	1	1.63	1.86

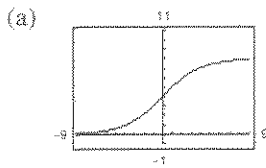
Asymptote:  $y = 2$ 

44.  $g(x) = 1 + e^{-x}$

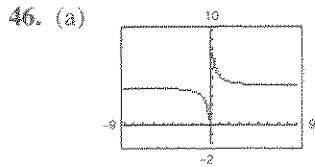
$x$	-2	-1	0	1	2
$g(x)$	8.39	3.72	2	1.37	1.14

Asymptote:  $y = 1$

45.  $f(x) = \frac{8}{1 + e^{-0.5x}}$


 (b)
 

$x$	-30	-20	-10	0	10	20	30
$f(x)$	$\approx 0$	$\approx 0$	0.05	4	7.95	$\approx 8$	$\approx 8$

 Horizontal asymptotes:  $y = 0, y = 8$ 

 (b)
 

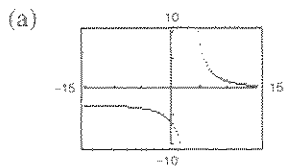
$x$	-15	-2	-1	-0.2	-0.1
$f(x)$	3.93	3.5	3.0	0.61	0.05

$x$	0	0.01	0.2	1	5
$f(x)$	undef.	8	7.4	5.0	4.2

 Horizontal asymptote:  $y = 4$ 

 Vertical asymptote:  $x = 0$ 

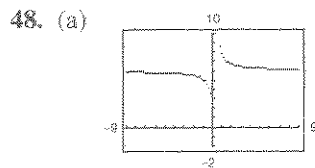
47.  $f(x) = \frac{-6}{2 - e^{0.2x}}$


 (b)
 

$x$	-20	-10	0	3	3.4	3.46
$f(x)$	-3.03	-3.22	-6	-34	-230	-2617

$x$	3.47	4	5	10	20
$f(x)$	3516	26.6	8.4	1.11	0.11

 Horizontal asymptotes:  $y = -3, y = 0$ 

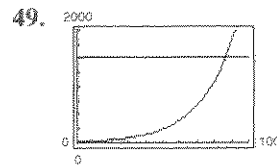
 Vertical asymptote:  $x \approx 3.47$ 

 (b)
 

$x$	-15	-10	-1	-0.1	-0.01
$f(x)$	5.9	5.9	5.1	3.2	3

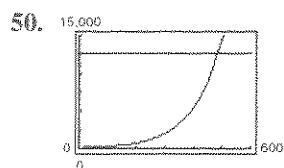
$x$	$\frac{0.2}{\ln 2}$	0.289	1	4	10
$f(x)$	undef.	2715	7.7	6.3	6.1

 Asymptotes:  $y = 6$ 

$$x = \frac{0.2}{\ln 2} \approx 0.2885$$

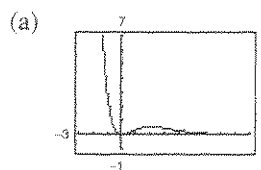


Intersection: (86.350, 1500)



Intersection: (482.831, 12,500)

51.  $f(x) = x^2 e^{-x}$



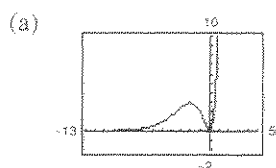
(b) Decreasing:  $(-\infty, 0)$ ,  $(2, \infty)$

Increasing:  $(0, 2)$

(c) Relative maximum:  $(2, 4e^{-2}) \approx (2, 0.541)$

Relative minimum:  $(0, 0)$

52.  $f(x) = 2x^2 e^{x+1}$



(b) Increasing on  $(-\infty, -2)$  and  $(0, \infty)$

Decreasing on  $(-2, 0)$

(c) Relative maximum:  $(-2, 2.943)$

Relative minimum:  $(0, 0)$

53.  $P = 2500$ ,  $r = 2.5\% = 0.025$ ,  $t = 10$

Compounded  $n$  times per year:  $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.025}{n}\right)^{10n}$

Compounded continuously:  $A = Pe^{rt} = 2500e^{(0.025)(10)}$

$n$	1	2	4	12	365	Continuous
$A$	3200.21	3205.09	3207.57	3209.23	3210.04	3210.06

54.  $P = 1000$ ,  $r = 6\% = 0.06$ ,  $t = 10$

$n$	1	2	4	12	365	Continuous
$A$	1790.85	1806.11	1814.02	1819.40	1822.03	1822.12

55.  $P = 2500$ ,  $r = 4\% = 0.04$ ,  $t = 20$

Compounded  $n$  times per year:  $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.04}{n}\right)^{20n}$

Compounded continuously:  $A = Pe^{rt} = 2500e^{(0.04)(20)}$

$n$	1	2	4	12	365	Continuous
$A$	5477.81	5520.10	5541.79	5556.46	5563.61	5563.85

56.  $P = 1000$ ,  $r = 3\% = 0.03$ ,  $t = 40$

$n$	1	2	4	12	365	Continuous
$A$	3262.04	3290.66	3305.28	3315.15	3319.95	3320.12



57.  $P = 12,000, r = 4\% = 0.04$

$$A = Pe^{rt} = 12000e^{0.04t}$$

$t$	1	10	20	30	40	50
$A$	12,489.73	17,901.90	26,706.49	39,841.40	59,436.39	88,668.67

58.  $P = 12,000, r = 6\% = 0.06$ , compounded continuously:  $A = Pe^{rt} = 12,000e^{(0.06)t}$

$t$	1	10	20	30	40	50
$A$	12,742.04	21,865.43	39,841.40	75,595.77	132,278.12	241,026.44

59.  $P = 12,000, r = 3.5\% = 0.035$

$$A = Pe^{rt} = 12,000e^{0.035t}$$

$t$	1	10	20	30	40	50
$A$	12,427.44	17,028.81	24,165.03	34,291.81	48,662.40	69,055.23

60.  $P = 12,000, r = 2.5\% = 0.025$ , compounded continuously:  $A = Pe^{rt} = 12,000e^{0.025t}$

$t$	1	10	20	30	40	50
$A$	12,303.78	15,408.31	19,784.66	25,404.00	32,619.38	41,884.12

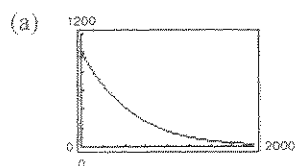
$$\begin{aligned} 61. A &= 25 \left[ \frac{(1 + 0.12/12)^{48} - 1}{0.12/12} \right] \\ &= 25 \left[ \frac{1.01^{48} - 1}{0.01} \right] \\ &= \$1530.57 \end{aligned}$$

$$\begin{aligned} 62. A &= 100 \left[ \frac{(1 + 0.09/12)^{60} - 1}{0.09/12} \right] \\ &= \$7542.41 \end{aligned}$$

$$\begin{aligned} 63. A &= 200 \left[ \frac{(1 + 0.06/12)^{72} - 1}{0.06/12} \right] \\ &= \$17,281.77 \end{aligned}$$

$$\begin{aligned} 64. A &= 75 \left[ \frac{(1 + 0.03/12)^{24} - 1}{0.03/12} \right] \\ &= \$1852.71 \end{aligned}$$

65.  $p = 5000 \left( 1 - \frac{4}{4 + e^{-0.002x}} \right)$



(b) If  $x = 500, p \approx \$421.12$ .

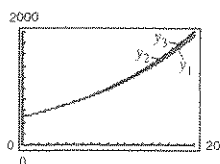
(c) For  $x = 600, p \approx \$350.13$ .

$x$	100	200	300	400	500	600	700
$p$	849.53	717.64	603.25	504.94	421.12	350.13	290.35

66. (a)  $y_1 = 500(1 + 0.07)^x$

$$y_2 = 500\left(1 + \frac{0.07}{4}\right)^{4x}$$

$$y_3 = 500e^{0.07x}$$

(b)  $y_3$  has the highest return.

After 20 years,

$$y_2 - y_1 = 2003.20 - 1934.84 = \$68.36$$

$$y_3 - y_2 = 2027.60 - 2003.20 = \$24.40$$

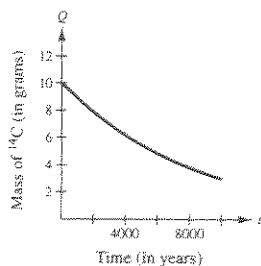
$$y_3 - y_1 = 2027.60 - 1934.84 = \$92.76$$

68.  $Q = 10\left(\frac{1}{2}\right)^{t/5715}$

(a) When  $t = 0$ ,  $Q = 10$ .(b) When  $t = 2000$ ,

$$Q = 10\left(\frac{1}{2}\right)^{2000/5715} \approx 7.85 \text{ grams.}$$

(c)



67.  $Q = 25\left(\frac{1}{2}\right)^{t/1599}$

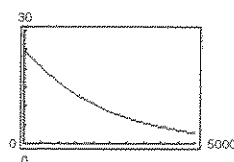
(a) When  $t = 0$ ,

$$Q = 25\left(\frac{1}{2}\right)^{0/1599} = 25(1) = 25 \text{ grams.}$$

(b) When  $t = 1000$ ,

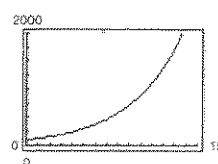
$$Q = 25\left(\frac{1}{2}\right)^{1000/1599} \approx 16.21 \text{ grams.}$$

(c)

(d) Never. The graph has a horizontal asymptote  $Q = 0$ .

69.  $P(t) = 100e^{0.2197t}$

(a)

(b)  $P(0) = 100$ 

$$P(5) \approx 300$$

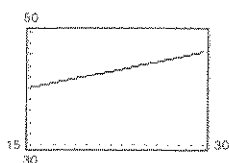
$$P(10) \approx 900$$

(c)  $P(0) = 100e^{0.2197(0)} = 100$ 

$$P(5) = 100e^{0.2197(5)} = 299.966 \approx 300$$

$$P(10) = 100e^{0.2197(10)} = 899.798 \approx 900$$

70. (a)



(b)

Year	2015	2016	2017	2018	2019	2020	2021	2022
$P$	40.1	40.5	40.9	41.3	41.7	42.1	42.5	43.0

Year	2023	2024	2025	2026	2027	2028	2029	2030
$P$	43.4	43.8	44.2	44.7	45.1	45.5	46.0	46.4

(c)  $P = 34.706e^{0.0097t} = 50$

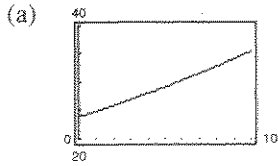
$$e^{0.0097t} = 1.441$$

$$0.0097t = \ln(1.441)$$

$$t \approx 38, \text{ or } 2038$$

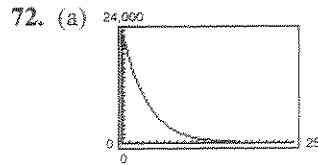
(Answers will vary.)

71.  $C(t) = P(1.04)^t$



(b)  $C(10) \approx 35.45$

(c)  $C(10) = 23.95(1.04)^{10} \approx 35.45$



(b)

$t$	1	2	3	4	5
$V$	17,978	13,483	10,112	7584	5688

$t$	6	7	8	9	10
$V$	4266	3200	2400	1800	1350

(c) According to the model,  $V(t) \rightarrow 0$  as  $t$  increases. However,  $V \neq 0$ .

 73. True.  $f(x) = 1^x$  is not an exponential function.

 74. False.  $e$  is an irrational number.

 75. The graph decreases for all  $x$  and has positive  $y$ -intercept. Matches (d).

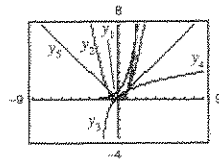
76.  $y_1 = e^x$

$y_2 = x^2$

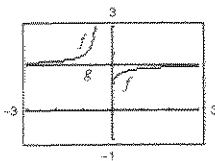
$y_3 = x^3$

$y_4 = \sqrt{x}$

$y_5 = |x|$



77.  $f(x) = \left(1 + \frac{0.5}{x}\right)^x$  and  $g(x) = e^{0.5} \approx 1.6487$   
 (Horizontal line)



As  $x \rightarrow \infty$ ,  $f(x) \rightarrow g(x)$ .

 (a)  $y_1 = e^x$  increases at the fastest rate.

 (b) For any positive integer  $n$ ,  $e^x > x^n$  for  $x$  sufficiently large. That is,  $e^x$  grows faster than  $x^n$ .

 (c) A quantity is growing exponentially if its growth rate is of the form  $y = ce^{rx}$ . This is a faster rate than any polynomial growth rate.

 78.  $y = 3^x$  (c) and  $y = 2^{-x}$  (d) are exponential functions because the exponents are variable.

79.  $e^\pi \approx 23.14$ ,  $\pi^e \approx 22.46$

$e^\pi > \pi^e$

81.  $5^{-3} = 0.008$ ,  $3^{-5} \approx 0.0041$

$5^{-3} > 3^{-5}$

80.  $2^{10} = 1024$ ,  $10^2 = 100$

$2^{10} > 10^2$

82.  $4^{1/2} = 2$ ,  $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$4^{1/2} > \left(\frac{1}{2}\right)^4$

83.  $f$  has an inverse because  $f$  is one-to-one.

$$y = 5x - 7$$

$$x = 5y - 7$$

$$x + 7 = 5y$$

$$f^{-1}(x) = \frac{1}{5}(x + 7)$$

84.  $f$  is one-to-one, so it has an inverse.

$$f(x) = -\frac{2}{3}x + \frac{5}{2}$$

$$y = -\frac{2}{3}x + \frac{5}{2}$$

$$x = -\frac{2}{3}y + \frac{5}{2}$$

$$x - \frac{5}{2} = -\frac{2}{3}y$$

$$-\frac{3}{2}\left(x - \frac{5}{2}\right) = y$$

$$f^{-1}(x) = -\frac{3}{2}x + \frac{15}{4}$$

85.  $f$  has an inverse because  $f$  is one-to-one.

$$y = \sqrt[3]{x + 8}$$

$$x = \sqrt[3]{y + 8}$$

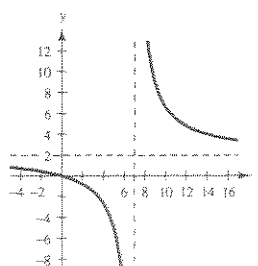
$$x^3 = y + 8$$

$$x^3 - 8 = y$$

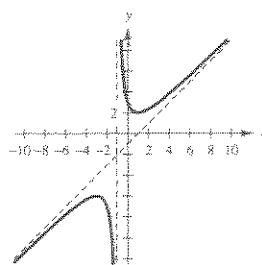
$$f^{-1}(x) = x^3 - 8$$

86.  $f$  is not one-to-one, so it does not have an inverse.

$$87. f(x) = \frac{2x}{x - 7}$$

Vertical asymptote:  $x = 7$ Horizontal asymptote:  $y = 2$ Intercept:  $(0, 0)$ 

$$88. f(x) = \frac{x^2 + 3}{x + 1} = x - 1 + \frac{4}{x + 1}$$

Slant asymptote:  $y = x - 1$ Vertical asymptote:  $x = -1$ Intercept:  $(0, 3)$ 

89. Answers will vary.

## Section 3.2 Logarithmic Functions and Their Graphs

■ You should know that a function of the form  $y = \log_a x$ , where  $a > 0$ ,  $a \neq 1$ , and  $x > 0$ , is called a logarithm of  $x$  to base  $a$ .

■ You should be able to convert from logarithmic form to exponential form and vice versa.

$$y = \log_a x \iff a^y = x$$

■ You should know the following properties of logarithms.

- (a)  $\log_a 1 = 0$  since  $a^0 = 1$ . (c)  $\log_a a^x = x$  since  $a^x = a^x$ .  
 (b)  $\log_a a = 1$  since  $a^1 = a$ . (d) If  $\log_a x = \log_a y$ , then  $x = y$ .

■ You should know the definition of the natural logarithmic function.

$$\log_e x = \ln x, x > 0$$

■ You should know the properties of the natural logarithmic function.

- (a)  $\ln 1 = 0$  since  $e^0 = 1$ . (c)  $\ln e^x = x$  since  $e^x = e^x$ .  
 (b)  $\ln e = 1$  since  $e^1 = e$ . (d) If  $\ln x = \ln y$ , then  $x = y$ .

■ You should be able to graph logarithmic functions.

## Vocabulary Check

1. logarithmic function      2. 10      3. natural logarithmic  
 4.  $a^{\log_a x} = x$       5.  $x = y$

1.  $\log_4 64 = 3 \implies 4^3 = 64$       2.  $\log_3 81 = 4 \implies 3^4 = 81$       3.  $\log_7 \frac{1}{49} = -2 \implies 7^{-2} = \frac{1}{49}$   
 4.  $\log_{10} \frac{1}{1000} = -3 \implies 10^{-3} = \frac{1}{1000}$       5.  $\log_{32} 4 = \frac{2}{5} \implies 32^{2/5} = 4$       6.  $\log_{16} 8 = \frac{3}{4} \implies 16^{3/4} = 8$   
 7.  $\ln 1 = 0 \implies e^0 = 1$       8.  $\ln 4 = 1.3862 \dots \implies e^{1.3862 \dots} = 4$       9.  $\ln e = 1 \implies e^1 = e$   
 10.  $\ln e^3 = 3 \implies e^3 = e^3$       11.  $\ln \sqrt{e} = \frac{1}{2} \implies e^{1/2} = \sqrt{e}$       12.  $\ln \frac{1}{e^2} = -2 \implies e^{-2} = \frac{1}{e^2}$   
 13.  $5^3 = 125 \implies \log_5 125 = 3$       14.  $8^2 = 64 \implies \log_8 64 = 2$       15.  $81^{1/4} = 3 \implies \log_{81} 3 = \frac{1}{4}$   
 16.  $9^{3/2} = 27 \implies \log_9 27 = \frac{3}{2}$       17.  $6^{-2} = \frac{1}{36} \implies \log_6 \frac{1}{36} = -2$   
 18.  $10^{-3} = 0.001 \implies \log_{10} 0.001 = -3$       19.  $e^3 = 20.0855 \dots \implies \ln 20.0855 \dots = 3$   
 20.  $e^4 \approx 54.5981 \dots \implies \ln 54.5981 \dots = 4$       21.  $e^{1.3} = 3.6692 \dots \implies \ln 3.6692 \dots = 1.3$   
 22.  $e^{2.5} = 12.1824 \dots \implies \ln 12.1824 \dots = 2.5$       23.  $\sqrt[3]{e} = 1.3956 \dots \implies \ln(1.3956 \dots) = \frac{1}{3}$

$$24. \frac{1}{e^4} = e^{-4} = 0.0183 \dots \Rightarrow \ln 0.0183 \dots = -4$$

$$25. \log_2 16 = \log_2 2^4 = 4$$

$$26. \log_{16} \left( \frac{1}{4} \right) = -\frac{1}{2}$$

$$\text{because } 16^{-1/2} = \frac{1}{16^{1/2}} = \frac{1}{4}$$

$$27. g \left( \frac{1}{1000} \right) = \log_{10} \left( \frac{1}{1000} \right)$$

$$= \log_{10} (10^{-3})$$

$$= -3$$

$$28. g(10,000) = \log_{10}(10,000)$$

$$= \log_{10}(10^4)$$

$$= 4$$

$$29. \log_{10} 345 \approx 2.538$$

$$30. \log_{10} \left( \frac{4}{5} \right) \approx -0.097$$

$$31. 6 \log_{10} 14.8 \approx 7.022$$

$$32. 1.9 \log_{10}(4.3) \approx 1.204$$

$$33. \log_7 x = \log_7 9$$

$$x = 9$$

$$34. x = \log_5 5 = 1$$

$$35. \log_6 6^2 = x$$

$$2 \log_6 6 = x$$

$$2 = x$$

$$36. \log_2 2^{-1} = x$$

$$-1 = x$$

$$37. \log_8 x = \log_8 10^{-1}$$

$$x = 10^{-1} = \frac{1}{10}$$

$$38. x = \log_4(4^3)$$

$$= 3$$

$$39. \log_4 4^{3x} = (3x) \log_4 4 = 3x$$

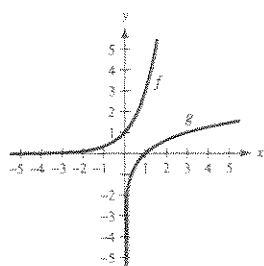
$$40. 6^{\log_6 36} = 36$$

$$41. 3 \log_2 \left( \frac{1}{2} \right) = 3 \log_2 (2^{-1})$$

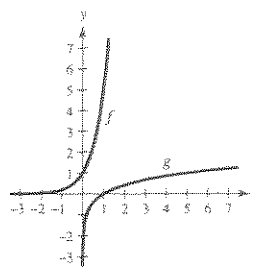
$$= 3(-1) = -3$$

$$42. \frac{1}{4} \log_4 16 = \frac{1}{4} \log_4 4^2 = \frac{1}{4}(2) = \frac{1}{2}$$

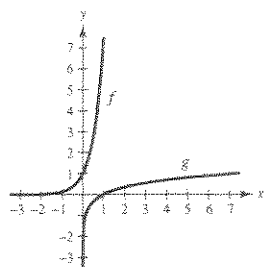
43.  $f(x) = 3^x$  and  $g(x) = \log_3 x$  are inverses of each other.



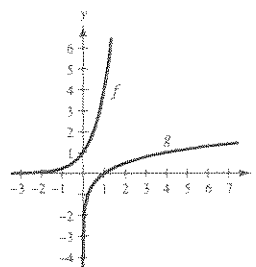
44.  $f(x) = 5^x$  and  $g(x) = \log_5 x$  are inverses of each other.



45.  $f(x) = e^{2x}$  and  $g(x) = \frac{1}{2} \ln x$  are inverses of each other.



46.  $f(x) = 4^x$  and  $g(x) = \log_4 x$  are inverses of each other.



47.  $y = \log_2(x + 2)$

Domain:  $x + 2 > 0 \Rightarrow x > -2$

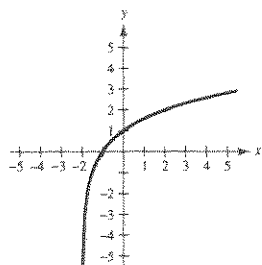
Vertical asymptote:  $x = -2$

$\log_2(x + 2) = 0$

$x + 2 = 1$

$x = -1$

x-intercept:  $(-1, 0)$



48.  $y = \log_2(x - 1)$

Domain:  $x - 1 > 0 \Rightarrow x > 1$

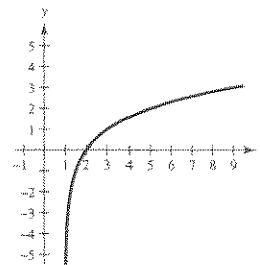
Vertical asymptote:  $x = 1$

$\log_2(x - 1) = 0$

$x - 1 = 1$

$x = 2$

x-intercept:  $(2, 0)$



49.  $y = 1 + \log_2 x$

Domain:  $x > 0$

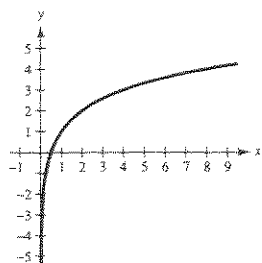
Vertical asymptote:  $x = 0$

$1 + \log_2 x = 0$

$\log_2 x = -1$

$x = 2^{-1} = \frac{1}{2}$

x-intercept:  $(\frac{1}{2}, 0)$



50.  $y = 2 - \log_2 x$

Domain:  $x > 0$

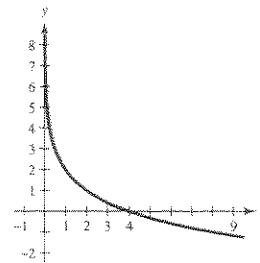
Vertical asymptote:  $x = 0$

$2 - \log_2 x = 0$

$\log_2 x = 2$

$x = 2^2 = 4$

x-intercept:  $(4, 0)$



51.  $y = 1 + \log_2(x - 2)$

Domain:  $x - 2 > 0 \Rightarrow x > 2$

Vertical asymptote:  $x = 2$

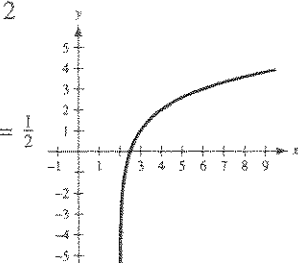
$1 + \log_2(x - 2) = 0$

$\log_2(x - 2) = -1$

$x - 2 = 2^{-1} = \frac{1}{2}$

$x = \frac{5}{2}$

x-intercept:  $(\frac{5}{2}, 0)$



52.  $y = 2 + \log_2(x + 1)$

Domain:  $x + 1 > 0 \Rightarrow x > -1$

Vertical asymptote:  $x = -1$

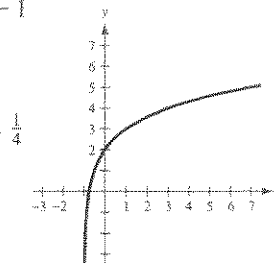
$2 + \log_2(x + 1) = 0$

$\log_2(x + 1) = -2$

$x + 1 = 2^{-2} = \frac{1}{4}$

$x = -\frac{3}{4}$

x-intercept:  $(-\frac{3}{4}, 0)$



53.  $f(x) = \log_3 x + 2$

Asymptote:  $x = 0$

Point on graph:  $(1, 2)$

Matches graph (b).

54.  $f(x) = -\log_3 x$

Asymptote:  $x = 0$

Point on graph:  $(1, 0)$

Matches graph (c).

55.  $f(x) = -\log_3(x + 2)$

Asymptote:  $x = -2$

Point on graph:  $(-1, 0)$

Matches graph (d).

56.  $f(x) = \log_3(1 - x)$

Asymptote:  $x = 1$ Domain:  $1 - x > 0 \Rightarrow x < 1$ Point on graph:  $(0, 0)$ 

Matches graph (a).

57.  $f(x) = \log_{10} x$

 $g(x) = -\log_{10} x$  is a reflection in the  $x$ -axis of the graph of  $f$ .

58. The graph of

 $g(x) = \log_{10}(x + 7)$  is a horizontal shift 7 units to the left of the graph of  $f(x) = \log_{10} x$ .

59.  $f(x) = \log_2 x$

 $g(x) = 4 - \log_2 x$  is obtained from  $f$  by a reflection in the  $x$ -axis followed by a vertical shift four units upward.60. The graph of  $g(x) = \log_2 x + 3$  is a vertical shift three units upward of the graph of  $f(x) = \log_2 x$ .

61. Horizontal shift three units to the left and a vertical shift two units downward

62. Horizontal shift one unit to the right and a vertical shift four units upward

63.  $\ln \sqrt{42} \approx 1.869$

64.  $\ln 18.31 \approx 2.907$

65.  $-\ln\left(\frac{1}{2}\right) \approx 0.693$

66.  $3 \ln(0.75) \approx -0.863$

67.  $\ln e^2 = 2$

68.  $-\ln e = -1$

69.  $e^{\ln 1.8} = 1.8$

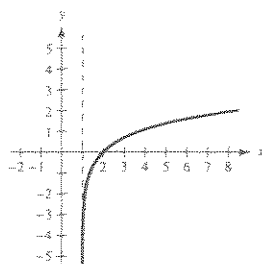
70.  $7 \ln e^0 = 7 \ln 1$

(Inverse Property)

(Inverse Property)

$= 7(0) = 0$

71.  $f(x) = \ln(x - 1)$

Domain:  $x > 1$ Vertical asymptote:  $x = 1$  $x$ -intercept:  $(2, 0)$ 

72.  $h(x) = \ln(x + 1)$

Domain:  $x + 1 > 0 \Rightarrow x > -1$ The domain is  $(-1, \infty)$ .Vertical asymptote:  $x + 1 = 0 \Rightarrow x = -1$  $x$ -intercept:  $\ln(x + 1) = 0$ 

$e^0 = x + 1$

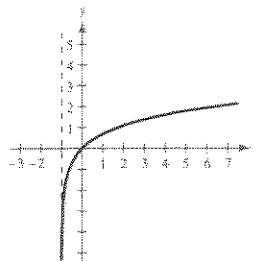
$1 = x + 1$

$0 = x$

The  $x$ -intercept is  $(0, 0)$ .

$y = \ln(x + 1) \Rightarrow e^y - 1 = x$

$x$	-0.39	0	1.72	6.39	19.09
$y$	$-\frac{1}{2}$	0	1	2	3





73.  $g(x) = \ln(-x)$

Domain:  $-x > 0 \Rightarrow x < 0$

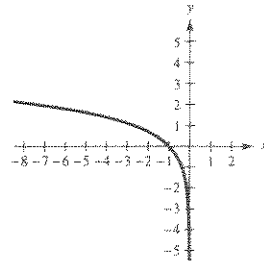
 The domain is  $(-\infty, 0)$ .

Vertical asymptote:  $-x = 0 \Rightarrow x = 0$

x-intercept:  $0 = \ln(-x)$

$$e^0 = -x$$

$$-1 = x$$

 The x-intercept is  $(-1, 0)$ .


74.  $f(x) = \ln(3 - x)$

Domain:  $3 - x > 0 \Rightarrow x < 3$

 The domain is  $(-\infty, 3)$ .

Vertical asymptote:  $3 - x = 0 \Rightarrow x = 3$

x-intercept:  $\ln(3 - x) = 0$

$$e^0 = 3 - x$$

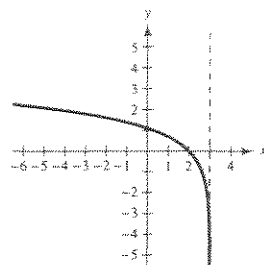
$$1 = 3 - x$$

$$2 = x$$

 The x-intercept is  $(2, 0)$ .

$$y = \ln(3 - x) \Rightarrow x = 3 - e^y$$

x	2.95	2.86	2.63	2	0.28
y	-3	-2	-1	0	1


 75.  $g(x) = \ln(x + 3)$  is a horizontal shift three units to the left.

 76.  $g(x) = \ln(x - 4)$  is a horizontal shift four units to the right.

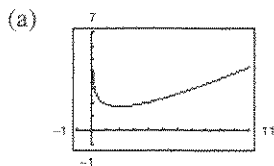
 77.  $g(x) = \ln x - 5$  is a vertical shift five units downward.

 78.  $g(x) = \ln x + 4$  is a vertical shift four units upward.

 79.  $g(x) = \ln(x - 1) + 2$  is a horizontal shift one unit to the right and a vertical shift two units upward.

 80.  $g(x) = \ln(x + 2) - 5$  is a horizontal shift two units to the left and a vertical shift five units downward.

81.  $f(x) = \frac{x}{2} - \ln \frac{x}{4}$

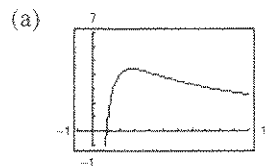

 (b) Domain:  $(0, \infty)$ 

 (c) Increasing on  $(2, \infty)$ 

 Decreasing on  $(0, 2)$ 

 (d) Relative minimum:  $(2, 1.693)$ 

82.  $g(x) = \frac{12 \ln x}{x}$

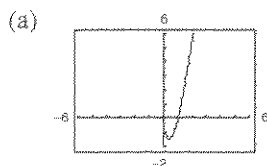

 (b) Domain:  $(0, \infty)$ 

 (c) Increasing on  $(0, 2.72)$ 

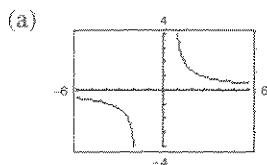
 Decreasing on  $(2.72, \infty)$ 

 (d) Relative maximum:  $(2.72, 4.41)$

83.  $h(x) = 4x \ln x$

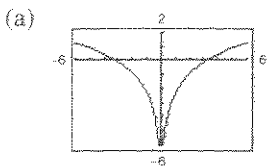
(b) Domain:  $(0, \infty)$ (c) Increasing on  $(0.368, \infty)$ Decreasing on  $(0, 0.368)$ (d) Relative minimum:  $(0.368, -1.472)$ 

85.  $f(x) = \ln\left(\frac{x+2}{x-1}\right)$

(b)  $\frac{x+2}{x-1} > 0$ ; Critical numbers:  $1, -2$ Test intervals:  $(-\infty, -2)$ ,  $(-2, 1)$ ,  $(1, \infty)$ Testing these three intervals, we see that the domain is  $(-\infty, -2) \cup (1, \infty)$ .(c) The graph is decreasing on  $(-\infty, -2)$  and decreasing on  $(1, \infty)$ .

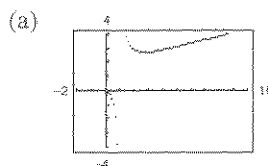
(d) There are no relative maximum or minimum values.

87.  $f(x) = \ln\left(\frac{x^2}{10}\right)$

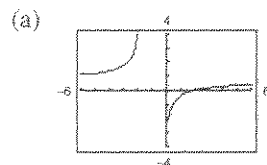
(b)  $\frac{x^2}{10} > 0 \Rightarrow x \neq 0$ ; Domain: all  $x \neq 0$ (c) The graph is increasing on  $(0, \infty)$  and decreasing on  $(-\infty, 0)$ .

(d) There are no relative maximum or relative minimum values.

84.  $f(x) = \frac{x}{\ln x}$

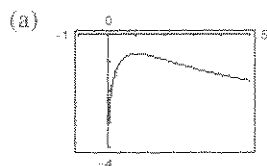
(b) Domain:  $(0, 1) \cup (1, \infty)$ (c) Increasing on  $(2.72, \infty)$ Decreasing on  $(0, 1) \cup (1, 2.72)$ (d) Relative minimum:  $(2.72, 2.72)$ 

86.  $f(x) = \ln\left(\frac{2x}{x+2}\right)$

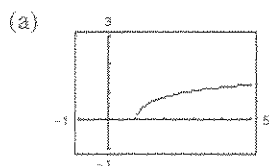
(b)  $\frac{2x}{x+2} > 0$ ; Critical numbers:  $0, -2$ Test intervals:  $(-\infty, -2)$ ,  $(-2, 0)$ ,  $(0, \infty)$ Testing these three intervals, we see that the domain is  $(-\infty, -2) \cup (0, \infty)$ .(c) The graph is increasing on  $(-\infty, -2)$  and increasing on  $(0, \infty)$ .

(d) There are no relative maximum or minimum values.

88.  $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

(b) Domain:  $x > 0$ (c) The graph is increasing on  $(0, 1)$  and decreasing on  $(1, \infty)$ .(d) Relative maximum:  $(1, -0.693)$

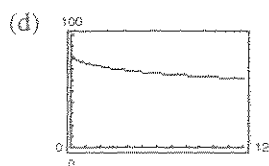
89.  $f(x) = \sqrt{\ln x}$



- (b)  $\ln x \geq 0 \Rightarrow x \geq 1$ ; Domain:  $x \geq 1$   
 (c) The graph is increasing on  $(1, \infty)$ .  
 (d) There are no relative maximum or relative minimum values.

91.  $f(t) = 80 - 17 \log_{10}(t + 1), \quad 0 \leq t \leq 12$

- (a)  $f(0) = 80 - 17 \log_{10}(0 + 1) = 80$   
 (b)  $f(4) = 80 - 17 \log_{10}(4 + 1) \approx 68.1$   
 (c)  $f(10) = 80 - 17 \log_{10}(10 + 1) \approx 62.3$

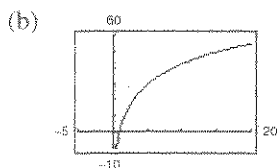


93.  $t = \frac{\ln K}{0.055}$

(a)

$K$	1	2	4	6	8	10	12
$t$	0	12.6	25.2	32.6	37.8	41.9	45.2

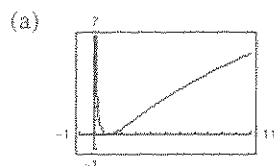
As the amount increases, the time increases, but at a lesser rate.



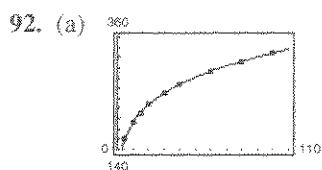
95.  $\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$

- (a)  $I = 1: \beta = 10 \log_{10}\left(\frac{1}{10^{-12}}\right) = 10 \cdot \log_{10}(10^{12}) = 10(12) = 120$  decibels  
 (b)  $I = 10^{-2}: \beta = 10 \log_{10}\left(\frac{10^{-2}}{10^{-12}}\right) = 10 \log_{10}(10^{10}) = 10(10) = 100$  decibels  
 (c) No, this is a logarithmic scale.

90.  $f(x) = (\ln x)^2$



- (b) Domain:  $x > 0$   
 (c) The graph is decreasing on  $(0, 1)$  and increasing on  $(1, \infty)$ .  
 (d) Relative minimum:  $(1, 0)$



The model is a good fit.

- (b)  $T > 300^\circ\text{F}$  when  $p > 67.3$  pounds per square inch

[The graph of  $T$  and  $y = 300$  intersect at  $p = 67.3$ .]

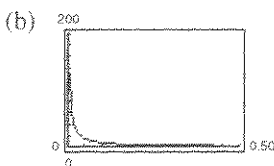
- (c)  $T(74) = 306.48^\circ\text{F}$

94. (a)

$r$	0.005	0.010	0.015
$t$	138.6 yr	69.3 yr	46.2 yr

$r$	0.020	0.025	0.030
$t$	34.7 yr	27.7 yr	23.1 yr

The doubling time decreases as  $r$  increases.



96.  $t = 16.625 \ln\left(\frac{x}{x - 750}\right), x > 750$

(a)  $16.625 \ln\left(\frac{897.72}{897.72 - 750}\right) \approx 30$  years

$16.625 \ln\left(\frac{1659.24}{1659.24 - 750}\right) \approx 10$  years

(b)  $(897.72)(30)(12) = 323,179.20$

$(1659.24)(10)(12) = 199,108.80$

Interest for 30-year loan is

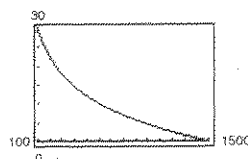
$323,179.20 - 150,000 = 173,179.20.$

Interest for 10-year loan is

$199,108.80 - 150,000 = 49,108.80.$

97.  $y = 80.4 - 11 \ln x$

$y(300) = 80.4 - 11 \ln 300 \approx 17.66 \text{ ft}^3/\text{min}$



98.  $y = 80.4 - 11 \ln x, 100 \leq x \leq 1500$

(a)  $\frac{450 \text{ cubic ft per minute}}{30 \text{ children}} = 15$  cubic feet per minute per child

(b) From the graph, for  $y = 15$  you get  $x \approx 382$  cubic feet.

(c) If ceiling height is 30, then 382 square feet of floor space is needed.

99. False. You would reflect  $y = 6^x$  in the line  $y = x$ .

100. True.  $\log_3(27) = \log_3 3^3 = 3$

101.  $5 = \log_b 32$

$b^5 = 32 = 2^5$

$b = 2$

102.  $4 = \log_b 81$

$b^4 = 81 = 3^4$

$b = 3$

103.  $2 = \log_b\left(\frac{1}{16}\right)$

$b^2 = \frac{1}{16} = \left(\frac{1}{4}\right)^2$

$b = \frac{1}{4}$

104.  $3 = \log_b\left(\frac{1}{27}\right)$

$b^3 = \frac{1}{27} = \left(\frac{1}{3}\right)^3$

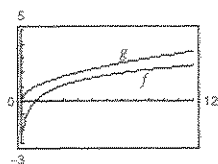
$b = \frac{1}{3}$

105. The vertical asymptote is to the right of the y-axis, and the graph increases. Matches (b).

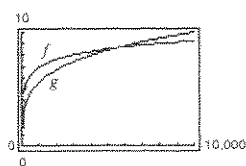
106. The vertical asymptote is to the left of the y-axis. Matches (b).

107.  $f(x) = \log_a x$  is the inverse of  $g(x) = a^x$ , where  $a > 0, a \neq 1$ .

108. (a)  $f(x) = \ln x, g(x) = \sqrt{x}$



(b)  $f(x) = \ln x, g(x) = \sqrt[4]{x}$



The rate of growth of the natural logarithmic function is slower than  $g(x) = x^{1/n}$  for any  $n$ .

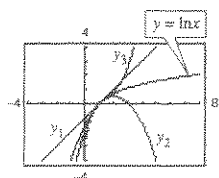
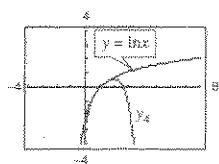
109. (a) False,  $y$  is not an exponential function of  $x$ . ( $y$  can never be 0.)

(b) True,  $y$  could be  $\log_2 x$ .

(c) True,  $x$  could be  $2^y$ .

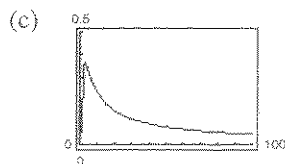
(d) False,  $y$  is not linear. (The points are not collinear.)

110. (a)


 (b) Pattern is  $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \dots$ 

 As you use more terms, the graph better approximates the graph of  $\ln x$  on the interval  $(0, 2)$ .

111.  $f(x) = \frac{\ln x}{x}$

(a)	$x$	1	5	10	$10^2$	$10^4$	$10^6$
	$f(x)$	0	0.322	0.230	0.046	0.00092	0.0000138

 (b) As  $x$  increases without bound,  $f(x)$  approaches 0.


112.  $f(t) = 75 - 6 \ln(t + 1)$

$$60 = 75 - 6 \ln(t + 1)$$

$$\ln(t + 1) = \frac{15}{6} = \frac{5}{2}$$

$$t = e^{5/2} - 1 \approx 11.18$$

 Or, you could graph  $f(t)$  and  $y = 60$  together in the same viewing window, and determine their point of intersection.

113.  $x^2 + 2x - 3 = (x + 3)(x - 1)$

114.  $2x^2 + 3x - 5 = (2x + 5)(x - 1)$

115.  $12x^2 + 5x - 3 = (4x + 3)(3x - 1)$

116.  $16x^2 + 16x + 7$

$$x = \frac{-16 \pm \sqrt{256 - 448}}{32}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{4}i$$

$$\left(x + \frac{1}{2} - \frac{\sqrt{3}}{4}i\right)\left(x + \frac{1}{2} + \frac{\sqrt{3}}{4}i\right)$$

117.  $16x^2 - 25 = (4x + 5)(4x - 5)$

118.  $36x^2 - 49 = (6x - 7)(6x + 7)$

$$119. 2x^3 + x^2 - 45x = x(2x^2 + x - 45) \\ = x(2x - 9)(x + 5)$$

$$120. 3x^3 - 5x^2 - 12x = x(3x^2 - 5x - 12) \\ = x(3x + 4)(x - 3)$$

$$121. (f + g)(2) = f(2) + g(2) = [3(2) + 2] + [2^3 - 1] = 8 + 7 = 15$$

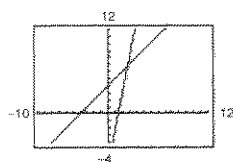
$$122. (f - g)(-1) = (-1) - (-2) = 1$$

$$123. (fg)(6) = f(6)g(6) = [3(6) + 2][6^3 - 1] = [20][215] = 4300$$

$$124. \left(\frac{f}{g}\right)(0) = \frac{2}{-1} = -2$$

$$125. 5x - 7 = x + 4$$

The graphs of  $y = 5x - 7$   
and  $y_2 = x + 4$   
intersect when  $x = 2.75$   
or  $\frac{11}{4}$ .



$$126. y = -2x + 3 \text{ and } y = 8x \text{ intersect at } x = 0.3$$

$$127. \sqrt{3x - 2} = 9$$

The graphs of  $y_1 = \sqrt{3x - 2}$  and  $y_2 = 9$  intersect  
when  $x \approx 27.667$  or  $\frac{83}{3}$ .

$$128. y = \sqrt{x - 11} \text{ and } y = x + 2 \text{ do not intersect.}$$

No solution

### Section 3.3 Properties of Logarithms

■ You should know the following properties of logarithms.

$$(a) \log_a x = \frac{\log_b x}{\log_b a}$$

$$(b) \log_a(uv) = \log_a u + \log_a v \quad \ln(uv) = \ln u + \ln v$$

$$(c) \log_a(u/v) = \log_a u - \log_a v \quad \ln(u/v) = \ln u - \ln v$$

$$(d) \log_a u^n = n \log_a u \quad \ln u^n = n \ln u$$

■ You should be able to rewrite logarithmic expressions using these properties.

#### Vocabulary Check

1. change-of-base

$$2. \frac{\ln x}{\ln a}$$

$$3. \log_a u^n$$

$$4. \ln u + \ln v$$

$$1. (a) \log_5 x = \frac{\log_{10} x}{\log_{10} 5}$$

$$2. (a) \log_3 x = \frac{\log_{10} x}{\log_{10} 3}$$

$$(b) \log_5 x = \frac{\ln x}{\ln 5}$$

$$(b) \log_3 x = \frac{\ln x}{\ln 3}$$

$$3. (a) \log_{1/5} x = \frac{\log_{10} x}{\log_{10} 1/5} = \frac{\log_{10} x}{-\log_{10} 5}$$

$$(b) \log_{1/5} x = \frac{\ln x}{\ln 1/5} = \frac{\ln x}{-\ln 5}$$

$$4. (a) \log_{1/3} x = \frac{\log_{10} x}{\log_{10} (1/3)} = \frac{-\log_{10} x}{\log_{10} 3}$$

$$(b) \log_{1/3} x = \frac{\ln x}{\ln (1/3)} = \frac{-\ln x}{\ln 3}$$

$$5. (a) \log_a \left( \frac{3}{10} \right) = \frac{\log_{10} (3/10)}{\log_{10} a}$$

$$(b) \log_a \left( \frac{3}{10} \right) = \frac{\ln (3/10)}{\ln a}$$

$$6. (a) \log_a \left( \frac{3}{4} \right) = \frac{\log_{10} (3/4)}{\log_{10} a}$$

$$(b) \log_a \left( \frac{3}{4} \right) = \frac{\ln (3/4)}{\ln a}$$

$$7. (a) \log_{2.6} x = \frac{\log_{10} x}{\log_{10} 2.6}$$

$$(b) \log_{2.6} x = \frac{\ln x}{\ln 2.6}$$

$$8. (a) \log_{7.1} x = \frac{\log_{10} x}{\log_{10} 7.1}$$

$$(b) \log_{7.1} x = \frac{\ln x}{\ln 7.1}$$

$$9. \log_3 7 = \frac{\ln 7}{\ln 3} \approx 1.771$$

$$10. \log_7 4 = \frac{\ln 4}{\ln 7} \approx 0.712$$

$$11. \log_{1/2} 4 = \frac{\ln 4}{\ln (1/2)} = -2$$

$$12. \log_{1/8} 64 = \frac{\ln 64}{\ln (1/8)} \\ = \frac{\ln 8^2}{-\ln 8} = -2$$

$$13. \log_9 (0.8) = \frac{\ln (0.8)}{\ln 9} \approx -0.102$$

$$14. \log_3 (0.015) = \frac{\ln (0.015)}{\ln 3} \\ \approx -3.823$$

$$15. \log_{15} 1460 = \frac{\ln 1460}{\ln 15} \approx 2.691$$

$$16. \log_{20} 135 = \frac{\ln 135}{\ln 20} \approx 1.637$$

$$17. \ln 20 = \ln (4 \cdot 5) \\ = \ln 4 + \ln 5$$

$$18. \ln 500 = \ln (5^3 \cdot 4) \\ = \ln 5^3 + \ln 4 \\ = 3 \ln 5 + \ln 4$$

$$19. \ln \frac{5}{64} = \ln 5 - \ln 64 \\ = \ln 5 - \ln 4^3 \\ = \ln 5 - 3 \ln 4$$

$$20. \ln \frac{2}{5} = \ln 2 - \ln 5 \\ = \ln 4^{1/2} - \ln 5 \\ = \frac{1}{2} \ln 4 - \ln 5$$

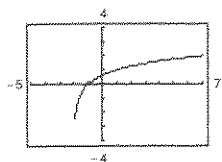
$$21. \log_b 25 = \log_b 5^2 \\ = 2 \log_b 5 \\ \approx 2(0.8271) \approx 1.6542$$

$$22. \log_b 30 = \log_b (2 \cdot 3 \cdot 5) \\ = \log_b 2 + \log_b 3 + \log_b 5 \\ \approx 0.3562 + 0.5646 + 0.8271 \approx 1.7479$$

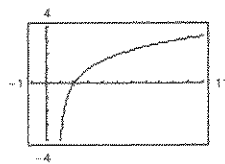
$$23. \log_b \sqrt{3} = \frac{1}{2} \log_b 3 \\ \approx \frac{1}{2}(0.5646) \\ \approx 0.2823$$

$$24. \log_b \left( \frac{25}{9} \right) = \log_b 5^2 - \log_b 3^2 \\ = 2 \log_b 5 - 2 \log_b 3 \\ \approx 2(0.8271) - 2(0.5646) \approx 0.5250$$

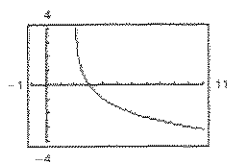
$$25. f(x) = \log_3(x+2) = \frac{\ln(x+2)}{\ln 3}$$



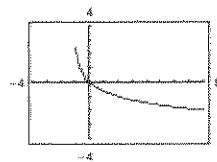
$$26. f(x) = \log_2(x-1) = \frac{\ln(x-1)}{\ln 2}$$



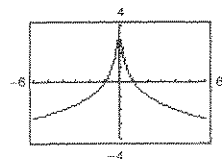
$$27. f(x) = \log_{1/2}(x-2) = \frac{\ln(x-2)}{\ln(1/2)} = \frac{\ln(x-2)}{-\ln 2}$$



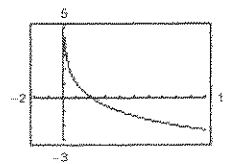
$$28. f(x) = \log_{1/3}(x+1) = \frac{\ln(x+1)}{\ln(1/3)} = \frac{\ln(x+1)}{-\ln 3}$$



$$29. f(x) = \log_{1/4}(x^2) = \frac{\ln x^2}{\ln(1/4)} = \frac{\ln x^2}{-\ln 4}$$



$$30. f(x) = \log_{1/2}\left(\frac{x}{2}\right) = \frac{\ln(x/2)}{\ln(1/2)} = \frac{\ln(x/2)}{-\ln 2}$$



$$\begin{aligned} 31. \log_4 8 &= \log_4 2^3 = 3 \log_4 2 \\ &= 3 \log_4 4^{1/2} = 3\left(\frac{1}{2}\right) \log_4 4 \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} 32. \log_2(4^2 \cdot 3^4) &= \log_2 4^2 + \log_2 3^4 \\ &= 2 \log_2 4 + 4 \log_2 3 \\ &= 2 \log_2 2^2 + 4 \log_2 3 \\ &= 4 \log_2 2 + 4 \log_2 3 \\ &= 4 + 4 \log_2 3 \end{aligned}$$

$$33. \ln(5e^6) = \ln 5 + \ln e^6 = \ln 5 + 6 = 6 + \ln 5$$

$$\begin{aligned} 34. \ln \frac{6}{e^2} &= \ln 6 - \ln e^2 \\ &= \ln 6 - 2 \ln e = \ln 6 - 2 \end{aligned}$$

$$\begin{aligned} 35. \log_5 \frac{1}{250} &= \log_5 1 - \log_5 250 = 0 - \log_5(125 \cdot 2) \\ &= -\log_5(5^3 \cdot 2) = -[\log_5 5^3 + \log_5 2] \\ &= -[3 \log_5 5 + \log_5 2] = -3 - \log_5 2 \end{aligned}$$

$$\begin{aligned} 36. -\ln 24 &= -\ln(2^3 \cdot 3) \\ &= -\ln 2^3 - \ln 3 \\ &= -3 \ln 2 - \ln 3 \\ &= -(3 \ln 2 + \ln 3) \end{aligned}$$

$$37. \log_{10} 5x = \log_{10} 5 + \log_{10} x$$

$$38. \log_{10} 10z = \log_{10} 10 + \log_{10} z = 1 + \log_{10} z$$

$$39. \log_{10} \frac{5}{x} = \log_{10} 5 - \log_{10} x$$

$$40. \log_{10} \frac{y}{2} = \log_{10} y - \log_{10} 2$$

$$41. \log_8 x^4 = 4 \log_8 x$$



42.  $\log_6 z^{-3} = -3 \log_6 z$

43.  $\ln \sqrt{z} = \ln z^{1/2} = \frac{1}{2} \ln z$

44.  $\ln \sqrt[3]{t} = \ln t^{1/3} = \frac{1}{3} \ln t$

45.  $\ln xyz = \ln x + \ln y + \ln z$

46.  $\ln \frac{xy}{z} = \ln x + \ln y - \ln z$

47.  $\log_3(a^2bc^3) = \log_3 a^2 + \log_3 b + \log_3 c^3$   
 $= 2 \log_3 a + \log_3 b + 3 \log_3 c$

48.  $\log_5(x^3y^3z) = \log_5 x^3 + \log_5 y^3 + \log_5 z$   
 $= 3 \log_5 x + 3 \log_5 y + \log_5 z$

49.  $\ln(a^2\sqrt{a-1}) = \ln a^2 + \ln(a-1)^{1/2}$   
 $= 2 \ln a + \frac{1}{2} \ln(a-1), a > 1$

50.  $\ln[z(z-1)^2] = \ln z + \ln(z-1)^2$   
 $= \ln z + 2 \ln(z-1)$

51.  $\ln \sqrt[3]{\frac{x}{y}} = \frac{1}{3} \ln \frac{x}{y}$   
 $= \frac{1}{3} [\ln x - \ln y]$   
 $= \frac{1}{3} \ln x - \frac{1}{3} \ln y$

52.  $\ln \sqrt{\frac{x^2}{y^3}} = \ln \left( \frac{x^2}{y^3} \right)^{1/2} = \frac{1}{2} \ln \left( \frac{x^2}{y^3} \right)$   
 $= \frac{1}{2} (\ln x^2 - \ln y^3)$   
 $= \frac{1}{2} (2 \ln x - 3 \ln y)$   
 $= \ln x - \frac{3}{2} \ln y$

53.  $\ln \left( \frac{x^2-1}{x^3} \right) = \ln(x^2-1) - \ln x^3$   
 $= \ln[(x-1)(x+1)] - 3 \ln x$   
 $= \ln(x-1) + \ln(x+1) - 3 \ln x, x > 1$

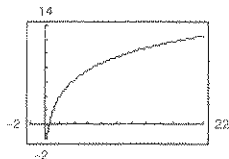
54.  $\ln \left( \frac{x}{\sqrt{x^2+1}} \right) = \ln x - \ln \sqrt{x^2+1}$   
 $= \ln x - \ln(x^2+1)^{1/2}$   
 $= \ln x - \frac{1}{2} \ln(x^2+1)$

55.  $\ln \left( \frac{x^4\sqrt{y}}{z^5} \right) = \ln x^4\sqrt{y} - \ln z^5$   
 $= \ln x^4 + \ln \sqrt{y} - \ln z^5$   
 $= 4 \ln x + \frac{1}{2} \ln y - 5 \ln z$

56.  $\log_b \frac{\sqrt{x}y^4}{z^4} = \log_b \sqrt{x}y^4 - \log_b z^4$   
 $= \log_b x^{1/2} + \log_b y^4 - \log_b z^4$   
 $= \frac{1}{2} \log_b x + 4 \log_b y - 4 \log_b z$

57.  $y_1 = \ln[x^3(x+4)]$   
 $y_2 = 3 \ln x + \ln(x+4)$

(a)



(b)

$x$	0.5	1	1.5	2	3	10
$y_1$	-0.5754	1.6094	2.9211	3.8712	5.2417	9.5468
$y_2$	-0.5754	1.6094	2.9211	3.8712	5.2417	9.5468

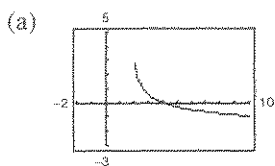
(c) The graphs and table suggest that

 $y_1 = y_2$  for  $x > 0$ . In fact,

$$y_1 = \ln[x^3(x+4)] = \ln x^3 + \ln(x+4)$$

$$= 3 \ln x + \ln(x+4) = y_2.$$

58.  $y_1 = \ln\left(\frac{\sqrt{x}}{x-2}\right)$ ,  $y_2 = \frac{1}{2} \ln x - \ln(x-2)$



(b)

$x$	3	4	5	6	10	20
$y_1$	0.5493	0	-0.2939	-0.4904	-0.9281	-1.393
$y_2$	0.5493	0	-0.2939	-0.4904	-0.9281	-1.393

(c) The graphs and table suggest that  $y_1 = y_2$ .

In fact,

$$y_1 = \ln\left(\frac{\sqrt{x}}{x-2}\right) = \ln x^{1/2} - \ln(x-2) = \frac{1}{2} \ln x - \ln(x-2) = y_2.$$

59.  $\ln x + \ln 4 = \ln 4x$

60.  $\ln y + \ln z = \ln yz$

61.  $\log_4 z - \log_4 y = \log_4 \frac{z}{y}$

62.  $\log_5 8 - \log_5 t = \log_5 \frac{8}{t}$

63.  $2 \log_2(x+3) = \log_2(x+3)^2$

64.  $\frac{5}{2} \log_7(z-4) = \log_7(z-4)^{5/2}$

65.  $\frac{1}{2} \ln(x^2 + 4) = \ln(x^2 + 4)^{1/2}$   
 $= \ln \sqrt{x^2 + 4}$

66.  $2 \ln x + \ln(x+1) = \ln x^2 + \ln(x+1)$   
 $= \ln(x^2(x+1))$   
 $= \ln(x^3 + x^2)$

67.  $\ln x - 3 \ln(x+1) = \ln x - \ln(x+1)^3$   
 $= \ln \frac{x}{(x+1)^3}$

68.  $\ln x - 2 \ln(x+2) = \ln x - \ln(x+2)^2$   
 $= \ln \frac{x}{(x+2)^2}$

69.  $\ln(x-2) - \ln(x+2) = \ln\left(\frac{x-2}{x+2}\right)$

70.  $3 \ln x + 2 \ln y - 4 \ln z = \ln x^3 + \ln y^2 - \ln z^4$   
 $= \ln x^3 y^2 - \ln z^4$   
 $= \ln \frac{x^3 y^2}{z^4}$

71.  $\ln x - 2[\ln(x+2) + \ln(x-2)] = \ln x - 2 \ln[(x+2)(x-2)]$   
 $= \ln x - 2 \ln(x^2 - 4)$   
 $= \ln x - \ln(x^2 - 4)^2$   
 $= \ln \frac{x}{(x^2 - 4)^2}$

72.  $4[\ln z + \ln(z+5)] - 2 \ln(z-5) = 4[\ln z(x+5)] - \ln(z-5)^2$   
 $= \ln[z(z+5)]^4 - \ln(z-5)^2$   
 $= \ln \frac{z^4(z+5)^4}{(z-5)^2}$

$$\begin{aligned}
 73. \quad \frac{1}{3}[2 \ln(x+3) + \ln x - \ln(x^2-1)] &= \frac{1}{3}[\ln(x+3)^2 + \ln x - \ln(x^2-1)] \\
 &= \frac{1}{3}[\ln[x(x+3)^2] - \ln(x^2-1)] \\
 &= \frac{1}{3} \ln \frac{x(x+3)^2}{x^2-1} \\
 &= \ln \sqrt[3]{\frac{x(x+3)^2}{x^2-1}}
 \end{aligned}$$

$$\begin{aligned}
 74. \quad 2[\ln x - \ln(x+1) - \ln(x-1)] &= 2\left[\ln \frac{x}{x+1} - \ln(x-1)\right] \\
 &= 2\left[\ln \frac{x}{(x+1)(x-1)}\right] \\
 &= 2\left[\ln \frac{x}{x^2-1}\right] \\
 &= \ln\left(\frac{x}{x^2-1}\right)^2
 \end{aligned}$$

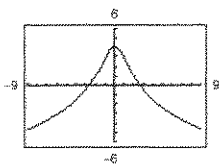
$$\begin{aligned}
 75. \quad \frac{1}{3}[\ln y + 2 \ln(y+4)] - \ln(y-1) &= \frac{1}{3}[\ln y + \ln(y+4)^2] - \ln(y-1) \\
 &= \frac{1}{3} \ln[y(y+4)^2] - \ln(y-1) \\
 &= \ln \sqrt[3]{y(y+4)^2} - \ln(y-1) \\
 &= \ln \frac{\sqrt[3]{y(y+4)^2}}{y-1}
 \end{aligned}$$

$$\begin{aligned}
 76. \quad \frac{1}{2}[\ln(x+1) + 2 \ln(x-1)] + 3 \ln x &= \frac{1}{2}[\ln(x+1) + \ln(x-1)^2] + \ln x^3 \\
 &= \frac{1}{2}[\ln(x+1)(x-1)^2] + \ln x^3 \\
 &= \ln[(x+1)(x-1)^2]^{1/2} + \ln x^3 \\
 &= \ln[(x+1)^{1/2}(x-1)] + \ln x^3 \\
 &= \ln[x^3(x-1)\sqrt{x+1}]
 \end{aligned}$$

$$77. \quad y_1 = 2[\ln 8 - \ln(x^2 + 1)]$$

$$y_2 = \ln \left[ \frac{64}{(x^2 + 1)^2} \right]$$

(a)

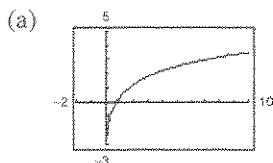
(c) The graphs and table suggest that  $y_1 = y_2$ . In fact,

$$\begin{aligned}
 y_1 &= 2[\ln 8 - \ln(x^2 + 1)] \\
 &= 2 \ln \frac{8}{x^2 + 1} = \ln \frac{64}{(x^2 + 1)^2} = y_2.
 \end{aligned}$$

(b)

$x$	-8	-4	-2	0	2	4	8
$y_1$	-4.1899	-1.5075	0.9400	4.1589	0.9400	-1.5075	-4.1899
$y_2$	-4.1899	-1.5075	0.9400	4.1589	0.9400	-1.5075	-4.1899

78.  $y_1 = \ln x + \frac{1}{2}\ln(x+1)$ ,  $y_2 = \ln(x\sqrt{x+1})$ ,  $x > 0$



(b)

$x$	0	1	2	5	10
$y_1$	ERROR	0.34657	1.2425	2.5053	3.5015
$y_2$	ERROR	0.34657	1.2425	2.5053	3.5015

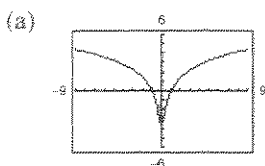
(c) The graphs and table suggest that  $y_1 = y_2$ .

In fact,

$$y_1 = \ln x + \frac{1}{2}\ln(x+1) = \ln x + \ln(x+1)^{1/2} = \ln[x\sqrt{x+1}] = y_2.$$

79.  $y_1 = \ln x^2$

$y_2 = 2 \ln x$



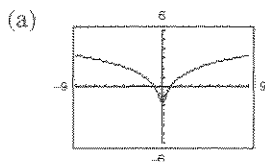
(The domain of  $y_2$  is  $x > 0$ .)

(b)

$x$	-8	-4	1	2	4
$y_1$	4.1589	2.7726	0	1.3863	2.7726
$y_2$	undefined	undefined	0	1.3863	2.7726

(c) The graphs and table suggest that  $y_1 = y_2$  for  $x > 0$ . The functions are not equivalent because the domains are different.

80.  $y_1 = \frac{1}{4}\ln[x^4(x^2+1)]$ ,  $y_2 = \ln x + \frac{1}{4}\ln(x^2+1)$



(b)

$x$	-10	-1	0	1	5	10
$y_1$	3.4564	0.17329	ERROR	0.17329	2.4240	3.4564
$y_2$	ERROR	ERROR	ERROR	0.17329	2.4240	3.4564

(c) No, the expressions are not equivalent. The domain of  $y_1$  is all  $x \neq 0$ , whereas the domain of  $y_2$  is  $x > 0$ .

81.  $\log_3 9 = 2 \log_3 3 = 2$

82.  $\log_6 \sqrt[3]{6} = \log_6 6^{1/3} = \frac{1}{3} \log_6 6 = \frac{1}{3}(1) = \frac{1}{3}$

83.  $\log_4 16^{3.4} = 3.4 \log_4(4^2) = 6.8 \log_4 4 = 6.8$

84.  $\log_5 \frac{1}{125} = \log_5 5^{-3} = -3 \log_5 5 = -3(1) = -3$

85.  $\log_2(-4)$  is undefined.  $-4$  is not in the domain of  $f(x) = \log_2 x$ .

86.  $\log_4(-16)$  is undefined because  $-16$  is not in the domain of  $\log_4 x$ .

87.  $\log_5 75 - \log_5 3 = \log_5 \frac{75}{3} = \log_5 25 = \log_5 5^2 = 2$

$$\begin{aligned} 88. \log_4 2 + \log_4 32 &= \log_4 4^{1/2} + \log_4 4^{5/2} \\ &= \frac{1}{2} \log_4 4 + \frac{5}{2} \log_4 4 \\ &= \frac{1}{2}(1) + \frac{5}{2}(1) \\ &= 3 \end{aligned}$$

89.  $\ln e^3 - \ln e^7 = 3 - 7 = -4$

90.  $\ln e^6 - 2 \ln e^5 = 6 \ln e - 10 \ln e = 6 - 10 = -4$

91.  $2 \ln e^4 = 2(4) \ln e = 8$

92.  $\ln e^{4.5} = 4.5 \ln e = 4.5$

$$93. \ln\left(\frac{1}{\sqrt{e}}\right) = \ln(1) - \ln e^{1/2} = 0 - \frac{1}{2} \ln e = -\frac{1}{2}$$

$$94. \ln \sqrt[5]{e^3} = \ln e^{3/5} = \frac{3}{5} \ln e = \frac{3}{5}$$

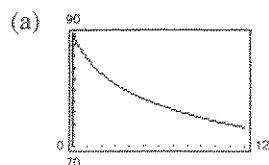
$$\begin{aligned} 95. (a) \beta &= 10 \cdot \log_{10}\left(\frac{I}{10^{-12}}\right) = 10[\log_{10} I - \log_{10} 10^{-12}] \\ &= 10[\log_{10} I - (-12) \log_{10} 10] \\ &= 10[\log_{10} I + 12] = 120 + 10 \cdot \log_{10} I \end{aligned}$$

(b)

$I$	$10^{-4}$	$10^{-6}$	$10^{-8}$	$10^{-10}$	$10^{-12}$	$10^{-14}$
$\beta$	80	60	40	20	0	-20

$$\begin{aligned} (c) \beta(10^{-4}) &= 120 + 10 \cdot \log_{10} 10^{-4} = 120 - 40 = 80 \\ \beta(10^{-6}) &= 120 + 10 \cdot \log_{10} 10^{-6} = 120 - 60 = 60 \\ \beta(10^{-8}) &= 120 + 10 \cdot \log_{10} 10^{-8} = 120 - 80 = 40 \\ \beta(10^{-10}) &= 120 + 10 \cdot \log_{10} 10^{-10} = 120 - 100 = 20 \\ \beta(10^{-12}) &= 120 + 10 \cdot \log_{10} 10^{-12} = 120 - 120 = 0 \\ \beta(10^{-14}) &= 120 + 10 \cdot \log_{10} 10^{-14} = 120 - 140 = -20 \end{aligned}$$

$$96. f(t) = 90 - 15 \log_{10}(t + 1), 0 \leq t \leq 12$$

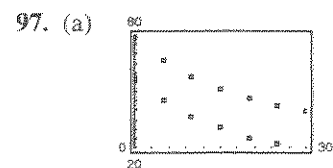


(b) When  $t = 0$ ,  $f(0) = 90$ .

(c)  $f(6) \approx 77$

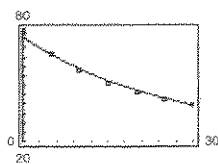
(d)  $f(12) \approx 73$

(e)  $f(t) = 75$  when  $t = 9$  months.



(b)  $T - 21 = 54.4(0.964)^t$

$$T = 21 + 54.4(0.964)^t$$



The data  $(t, T - 21)$  fits the model

$$T - 21 = 54.4(0.964)^t.$$

The model

$$T = 21 + 54.4(0.964)^t$$

fits the original data.

—CONTINUED—

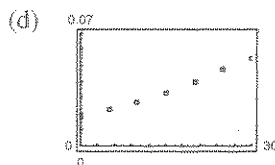
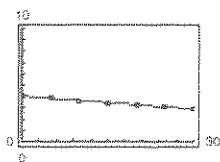
## 97. —CONTINUED—

(c)  $\ln(T - 21) = -0.0372t + 3.9971$ , linear model

$$T - 21 = e^{-0.0372t + 3.9971}$$

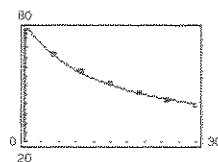
$$T = 21 + 54.4e^{-0.0372t}$$

$$= 21 + 54.4(0.964)^t$$

(d)  $\frac{1}{T - 21} = 0.00121t + 0.01615$ , linear model

$$T - 21 = \frac{1}{0.00121t + 0.01615}$$

$$T = 21 + \frac{1}{0.00121t + 0.01615}$$



98. If  $y = ab^x$ , then  $\ln y = \ln(ab^x) = \ln a + x \ln b$ , which is linear. If  $y = \frac{1}{cx + d}$ , then  $\frac{1}{y} = cx + d$ .

99. True

100. False. For example, let  $x = 2$  and  $a = 1$ . Then  $f(x - a) = \ln(2 - 1) = 0$ , but  $f(x) - f(a) = \ln(2) - \ln 1 = \ln 2$ .

101. False. For example, let  $x = 1$  and  $a = 2$ . Then  $f\left(\frac{x}{a}\right) = \ln\left(\frac{1}{2}\right)$ . But  $\frac{f(x)}{f(a)} = \frac{\ln 1}{\ln 2} = 0$ .

102. False. For example, let  $x = 1$  and  $a = 1$ . Then  $f(x + a) = \ln(1 + 1) = \ln 2$ , but  $f(x)f(a) = (\ln 1)(\ln 1) = 0$ .

103. False.  $\sqrt{\ln x} \neq \frac{1}{2} \ln x$   
In fact,  $\ln x^{1/2} = \frac{1}{2} \ln x$ .

104. False. For example, let  $n = 2$  and  $x = e$ . Then  $[f(x)]^n = [\ln e]^2 = 1$ , but  $nf(x) = 2 \ln e = 2$ .

105. True. In fact, if  $\ln x < 0$ , then  $0 < x < 1$ .

106. False. For example, let  $x = \sqrt{e}$ . Then  $f(x) = \ln \sqrt{e} = \frac{1}{2} \ln e = \frac{1}{2} > 0$ , but  $\sqrt{e} < e$ .

107. Let  $y = \log_a x$  and  $z = \log_{a/b} x$ , then  $a^y = x = \left(\frac{a}{b}\right)^z$  and

$$\left(\frac{1}{b}\right)^z = a^{y-z}$$

$$\frac{1}{b} = a^{(y-z)/z}$$

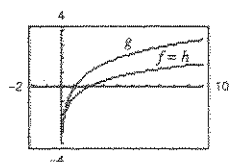
$$\log_a \left(\frac{1}{b}\right) = \frac{y-z}{z} = \frac{y}{z} - 1 \Rightarrow 1 + \log_a \left(\frac{1}{b}\right) = \frac{\log_a x}{\log_{a/b} x}$$

$$108. f(x) = \ln \frac{x}{2}$$

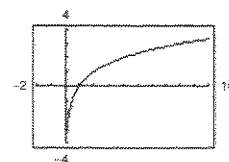
$$g(x) = \frac{\ln x}{\ln 2}$$

$$h(x) = \ln x - \ln 2$$

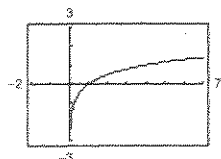
$$f(x) = h(x) \text{ by Property 2.}$$



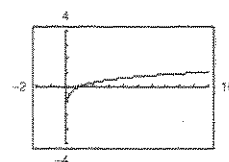
$$109. f(x) = \log_2 x = \frac{\ln x}{\ln 2}$$



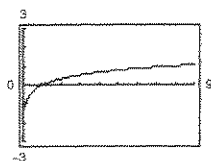
$$110. f(x) = \log_4 x = \frac{\ln x}{\ln 4}$$



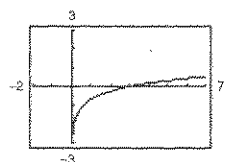
$$111. f(x) = \log_3 \sqrt{x} = \frac{1}{2} \frac{\ln x}{\ln 3}$$



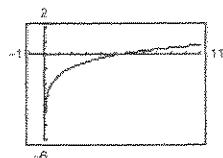
$$112. f(x) = \log_2 \sqrt[3]{x} = \frac{1}{3} \frac{\ln x}{\ln 2}$$



$$113. f(x) = \log_5 \left( \frac{x}{3} \right) = \frac{\ln(x/3)}{\ln 5}$$



$$114. f(x) = \log_3 \frac{x}{5} = \frac{\ln x - \ln 5}{\ln 3}$$



$$115. \ln 1 = 0, \ln 2 \approx 0.6931, \ln 3 \approx 1.0986, \ln 5 \approx 1.6094$$

$$\ln 2 \approx 0.6931$$

$$\ln 3 \approx 1.0986$$

$$\ln 4 = \ln 2 + \ln 2 \approx 0.6931 + 0.6931 = 1.3862$$

$$\ln 5 \approx 1.6094$$

$$\ln 6 = \ln 2 + \ln 3 \approx 0.6931 + 1.0986 = 1.7917$$

$$\ln 8 = \ln 2^3 = 3 \ln 2 \approx 3(0.6931) = 2.0793$$

$$\ln 9 = \ln 3^2 = 2 \ln 3 \approx 2(1.0986) = 2.1972$$

$$\ln 10 = \ln 5 + \ln 2 \approx 1.6094 + 0.6931 = 2.3025$$

$$\ln 12 = \ln 2^2 + \ln 3 = 2 \ln 2 + \ln 3 \approx 2(0.6931) + 1.0986 = 2.4848$$

$$\ln 15 = \ln 5 + \ln 3 \approx 1.6094 + 1.0986 = 2.7080$$

$$\ln 16 = \ln 2^4 = 4 \ln 2 \approx 4(0.6931) = 2.7724$$

$$\ln 18 = \ln 3^2 + \ln 2 = 2 \ln 3 + \ln 2 \approx 2(1.0986) + 0.6931 = 2.8903$$

$$\ln 20 = \ln 5 + \ln 2^2 = \ln 5 + 2 \ln 2 \approx 1.6094 + 2(0.6931) = 2.9956$$

$$116. \frac{24xy^{-2}}{16x^{-3}y} = \frac{24xx^3}{16yy^2} = \frac{3x^4}{2y^3}$$

$$117. \left(\frac{2x^2}{3y}\right)^{-3} = \left(\frac{3y}{2x^2}\right)^3$$

$$= \frac{(3y)^3}{(2x^2)^3}$$

$$= \frac{27y^3}{8x^6}$$

$$118. (18x^3y^4)^{-3}(18x^3y^4)^3 = \frac{(18x^3y^4)^3}{(18x^3y^4)^3} = 1 \text{ if } x \neq 0, y \neq 0$$

$$119. xy(x^{-1} + y^{-1})^{-1} = \frac{xy}{x^{-1} + y^{-1}}$$

$$= \frac{xy}{\frac{1}{x} + \frac{1}{y}}$$

$$= \frac{xy}{\frac{y+x}{xy}}$$

$$= \frac{(xy)^2}{x+y}, x \neq 0, y \neq 0$$

$$120. x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(2)}}{2} = 3 \pm \sqrt{7}$$

$$121. 2x^3 + 20x^2 + 50x = 0$$

$$2x(x^2 + 10x + 25) = 0$$

$$2x(x+5)^2 = 0$$

$$x = 0, -5, -5$$

$$122. x^4 - 19x^2 + 48 = 0$$

$$(x^2 - 16)(x^2 - 3) = 0$$

$$(x-4)(x+4)(x-\sqrt{3})(x+\sqrt{3}) = 0$$

$$x = \pm 4, \pm \sqrt{3}$$

$$123. 9x^4 - 37x^2 + 4 = 0$$

$$(x^2 - 4)(9x^2 - 1) = 0$$

$$(x-2)(x+2)(3x-1)(3x+1) = 0$$

$$x = \pm 2, \pm \frac{1}{3}$$

$$124. x^3 - 6x^2 - 4x + 24 = 0$$

$$x^2(x-6) - 4(x-6) = 0$$

$$(x^2 - 4)(x-6) = 0$$

$$(x-2)(x+2)(x-6) = 0$$

$$x = 2, -2, 6$$

$$125. 9x^4 - 226x^2 + 25 = 0$$

$$(x^2 - 25)(9x^2 - 1) = 0$$

$$(x-5)(x+5)(3x+1)(3x-1) = 0$$

$$x = \pm 5, \pm \frac{1}{3}$$



## Section 3.4 Solving Exponential and Logarithmic Equations

- To solve an exponential equation, isolate the exponential expression, then take the logarithm of both sides. Then solve for the variable.

1.  $\log_a a^x = x$

2.  $\ln e^x = x$

- To solve a logarithmic equation, rewrite it in exponential form. Then solve for the variable.

1.  $a^{\log_a x} = x$

2.  $e^{\ln x} = x$

- If  $a > 0$  and  $a \neq 1$  we have the following:

1.  $\log_a x = \log_a y \Rightarrow x = y$

2.  $a^x = a^y \Rightarrow x = y$

- Use your graphing utility to approximate solutions.

## Vocabulary Check

1. solve

2. (a)  $x = y$  (b)  $x = y$  (c)  $x$  (d)  $x$ 

3. extraneous

1.  $4^{2x-7} = 64$

(a)  $x = 5$

$4^{2(5)-7} = 4^3 = 64$

Yes,  $x = 5$  is a solution.

(b)  $x = 2$

$4^{2(2)-7} = 4^{-3} = \frac{1}{64} \neq 64$

No,  $x = 2$  is not a solution.

2.  $2^{3x+1} = 32$

(a)  $x = -1$

$2^{3(-1)+1} = 2^{-2} = \frac{1}{4}$

No,  $x = -1$  is not a solution.

(b)  $x = 2$

$2^{3(2)+1} = 2^7 = 128$

No,  $x = 2$  is not a solution.

3.  $3e^{x+2} = 75$

(a)  $x = -2 + e^{25}$

$3e^{(-2+e^{25})+2} = 3e^{e^{25}} \neq 75$

No,  $x = -2 + e^{25}$  is not a solution.

(b)  $x = -2 + \ln 25$

$3e^{(-2+\ln 25)+2} = 3e^{\ln 25} = 3(25) = 75$

Yes,  $x = -2 + \ln 25$  is a solution.

(c)  $x \approx 1.2189$

$3e^{1.2189+2} = 3e^{3.2189} \approx 75$

Yes,  $x \approx 1.2189$  is a solution.

4.  $4e^{x-1} = 60$

(a)  $x = 1 + \ln 15$

$4e^{(1+\ln 15)-1} = 4e^{\ln 15} = 4(15) = 60$

Yes

(b)  $x \approx 3.7081$

$4e^{3.7081-1} = 4e^{2.7081} \approx 60$

Yes

(c)  $x = \ln 16$

$4e^{\ln 16-1} \approx 23.5 \neq 60$

No

5.  $\log_4(3x) = 3$

$4^3 = 3x$

$x = \frac{64}{3} \approx 21.333$

(a)  $x = 21.3560$  is an approximate solution.(b) No,  $x = -4$  is not a solution.(c) Yes,  $x = \frac{64}{3}$  is a solution.

6.  $\log_6\left(\frac{5}{3}x\right) = 2 \Leftrightarrow \frac{\ln\left(\frac{5}{3}x\right)}{\ln 6} = 2$

(a)  $x \approx 20.2882$ ;  $\frac{\ln\left(\frac{5}{3} \cdot 20.2882\right)}{\ln 6} = 1.965 \neq 2$ ; No

(b)  $x = \frac{108}{5}$ ;  $\log_6\left(\frac{5}{3} \cdot \frac{108}{5}\right) = \log_6(36) = 2$ ; Yes

(c)  $x = 7.2$ ;  $\frac{\ln\left(\frac{5}{3}(7.2)\right)}{\ln 6} \approx 1.3869 \neq 2$ ; No

7.  $\ln(x - 1) = 3.8$

(a)  $x = 1 + e^{3.8}$

$\ln(1 + e^{3.8} - 1) = \ln e^{3.8} = 3.8$

Yes,  $x = 1 + e^{3.8}$  is a solution.

(c)  $x = 1 + \ln 3.8$

$\ln(1 + \ln 3.8 - 1) = \ln(\ln 3.8) \approx 0.289$

No,  $x = 1 + \ln 3.8$  is not a solution.

(b)  $x \approx 45.7012$

$\ln(45.7012 - 1) = \ln(44.7012) \approx 3.8$

Yes,  $x \approx 45.7012$  is a solution.

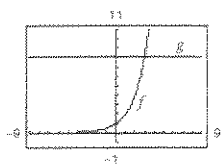
8.  $\ln(2 + x) = 2.5$

(a)  $x = e^{2.5} - 2$ ;  $\ln(2 + e^{2.5} - 2) = \ln e^{2.5} = 2.5$ ; Yes

(b)  $x \approx \frac{4073}{400}$ ;  $\ln\left(2 + \frac{4073}{400}\right) \approx 2.5$ ; Yes

(c)  $x = \frac{1}{2}$ ;  $\ln\left(2 + \frac{1}{2}\right) \approx 0.9163 \neq 2.5$ ; No

9.

Point of intersection:  $(3, 8)$ Algebraically:  $2^x = 8$ 

$2^x = 2^3$

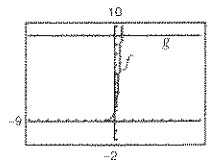
$x = 3 \Rightarrow y = 8 \Rightarrow (3, 8)$

10.  $f(x) = g(x)$

$27^x = 9$

$27^x = 27^{2/3}$

$x = \frac{2}{3}$

Point of intersection:  $\left(\frac{2}{3}, 9\right)$ 

11. Point of intersection: (4, 10)

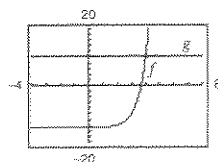
Algebraically:  $5^{x-2} - 15 = 10$

$$5^{x-2} = 25 = 5^2$$

$$x - 2 = 2$$

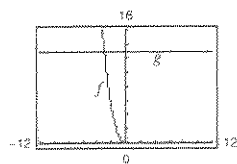
$$x = 4$$

(4, 10)



12.  $f(x) = 2^{-x+1} - 3$

$$g(x) = 13$$



Point of intersection: (-3, 13)

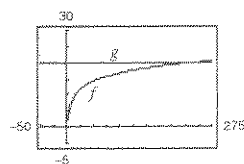
$$2^{-x+1} - 3 = 13$$

$$2^{-x+1} = 16 = 2^4$$

$$-x + 1 = 4$$

$$x = -3 \Rightarrow (-3, 13)$$

- 13.



Point of intersection: (243, 20)

Algebraically:  $4 \log_3 x = 20$

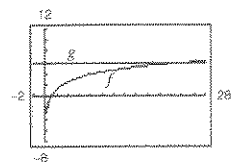
$$\log_3 x = 5$$

$$x = 3^5 = 243$$

(243, 20)

14.  $f(x) = 3 \log_5 x = 3 \cdot \frac{\ln x}{\ln 5}$

$$g(x) = 6$$



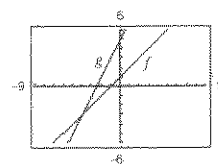
Point of intersection: (25, 6)

$$3 \log_5 x = 6$$

$$\log_5 x = 2$$

$$x = 5^2 = 25 \Rightarrow (25, 6)$$

- 15.



Point of intersection: (-4, -3)

Algebraically:  $\ln e^{x+1} = 2x + 5$

$$x + 1 = 2x + 5$$

$$-4 = x$$

(-4, -3)

16.  $f(x) = \ln e^{x-2} = x - 2$

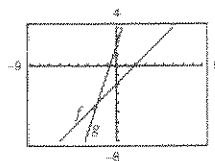
$$g(x) = 3x + 2$$

Point of intersection: (-2, -4)

$$x - 2 = 3x + 2$$

$$-4 = 2x$$

$$x = -2 \Rightarrow (-2, -4)$$



17.  $4^x = 16$

$4^x = 4^2$

$x = 2$

18.  $3^x = 243$

$3^x = 3^5$

$x = 5$

19.  $5^x = \frac{1}{625}$

$5^x = \frac{1}{5^4} = 5^{-4}$

$x = -4$

20.  $7^x = \frac{1}{49}$

$7^x = 7^{-2}$

$x = -2$

21.  $\left(\frac{1}{8}\right)^x = 64$

$8^{-x} = 8^2$

$-x = 2$

$x = -2$

22.  $\left(\frac{1}{2}\right)^x = 32$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5}$

$x = -5$

23.  $\left(\frac{2}{3}\right)^x = \frac{81}{16}$

$\left(\frac{2}{3}\right)^{-x} = \left(\frac{3}{2}\right)^4$

$-x = 4$

$x = -4$

24.  $\left(\frac{3}{4}\right)^x = \frac{27}{64}$

$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^3$

$x = 3$

25.  $6(10^x) = 216$

$10^x = 36$

$\log_{10} 10^x = \log_{10} 36$

$x = \log_{10} 36 \approx 1.5563$

26.  $5(8^x) = 325$

$8^x = 65$

$x = \log_8 65$

$= \frac{\ln 65}{\ln 8} \approx 2.0075$

27.  $2^{x+3} = 256$

$2^x \cdot 2^3 = 256$

$2^x = 32$

$x = 5$

*Alternate solution:*

$2^{x+3} = 2^8$

$x + 3 = 8$

$x = 5$

28.  $3^{x-1} = \frac{1}{81}$

$3^{x-1} = 3^{-4}$

$x - 1 = -4$

$x = -3$

29.  $\ln x - \ln 5 = 0$

$\ln x = \ln 5$

$x = 5$

30.  $\ln x - \ln 2 = 0$

$\ln x = \ln 2$

$x = 2$

31.  $\ln x = -7$

$x = e^{-7}$

32.  $\ln x = -1$

$e^{-1} = x$

$x = \frac{1}{e} \approx 0.368$

33.  $\log_x 625 = 4$

$x^4 = 625$

$x^4 = 5^4$

$x = 5$

34.  $\log_x 25 = 2$

$x^2 = 25$

$x = 5$

35.  $\log_{10} x = -1$

$x = 10^{-1}$

$x = \frac{1}{10}$

36.  $\log_{10} x = -\frac{1}{2}$

$x = 10^{-1/2} = \frac{1}{\sqrt{10}}$

$\approx 0.316$

37.  $\ln(2x - 1) = 5$

$2x - 1 = e^5$

$x = \frac{1 + e^5}{2} \approx 74.707$

38.  $\ln(3x + 5) = 8$

$e^8 = 3x + 5$

$x = \frac{1}{3}(e^8 - 5)$

$\approx 991.986$

39.  $\ln e^{x^2} = x^2 \ln e^x = x^2$

40.  $\ln e^{2x-1} = 2x - 1$

41.  $e^{\ln(5x+2)} = 5x + 2$

42.  $e^{\ln x^2} = x^2$

43.  $-1 + \ln e^{2x} = -1 + 2x = 2x - 1$

44.  $-8 + e^{\ln x^3} = -8 + x^3 = x^3 - 8$

45.  $8^{3x} = 360$

$$\ln 8^{3x} = \ln 360$$

$$3x \ln 8 = \ln 360$$

$$3x = \frac{\ln 360}{\ln 8}$$

$$x = \frac{1}{3} \frac{\ln 360}{\ln 8}$$

$$x \approx 0.944$$

46.  $6^{5x} = 3000$

$$\ln 6^{5x} = \ln 3000$$

$$(5x) \ln 6 = \ln 3000$$

$$5x = \frac{\ln 3000}{\ln 6}$$

$$x = \frac{\ln 3000}{5 \ln 6} \approx 0.894$$

47.  $5^{-t/2} = 0.20 = \frac{1}{5}$

$$-\frac{t}{2} \ln 5 = \ln\left(\frac{1}{5}\right)$$

$$-\frac{t}{2} \ln 5 = -\ln 5$$

$$\frac{t}{2} = 1$$

$$t = 2$$

48.  $4^{-3t} = 0.10$

$$\ln 4^{-3t} = \ln 0.10$$

$$(-3t) \ln 4 = \ln 0.10$$

$$-3t = \frac{\ln 0.10}{\ln 4}$$

$$t = -\frac{\ln 0.10}{3 \ln 4} \approx 0.554$$

49.  $5(2^{3-x}) - 13 = 100$

$$5(2^{3-x}) = 113$$

$$2^{3-x} = \frac{113}{5}$$

$$\ln 2^{3-x} = \ln\left(\frac{113}{5}\right)$$

$$3 - x = \frac{\ln(113/5)}{\ln 2}$$

$$x = 3 - \frac{\ln(113/5)}{\ln 2}$$

$$x \approx -1.498$$

50.  $6(8^{-2-x}) + 15 = 2601$

$$6(8^{-2-x}) = 2586$$

$$8^{-2-x} = 431$$

$$(-2-x) \ln 8 = \ln 431$$

$$-2-x = \frac{\ln 431}{\ln 8}$$

$$x = -2 - \frac{\ln 431}{\ln 8}$$

$$x \approx -4.917$$

51.  $\left(1 + \frac{0.10}{12}\right)^{12t} = 2$

$$\left(\frac{12.1}{12}\right)^{12t} = 2$$

$$(12t) \ln\left(\frac{12.1}{12}\right) = \ln 2$$

$$t = \frac{1}{12} \frac{\ln 2}{\ln(12.1/12)}$$

$$t \approx 6.960$$

52.  $\left(16 + \frac{0.878}{26}\right)^{3t} = 30$

$$3t \ln\left(16 + \frac{0.878}{26}\right) = \ln 30$$

$$t = \frac{\ln 30}{3 \ln\left(16 + \frac{0.878}{26}\right)}$$

$$\approx \frac{3.4012}{8.3241} \approx 0.409$$

53.  $5000 \left[ \frac{(1 + 0.005)^x}{0.005} \right] = 250,000$

$$5000(1.005)^x = 1250$$

$$1.005^x = 0.25$$

$$x \ln(1.005) = \ln 0.25$$

$$x = \frac{\ln 0.25}{\ln(1.005)}$$

$$x \approx -277.951$$

54.  $250 \left[ \frac{(1 + 0.01)^x}{0.01} \right] = 150,000$

$$250(1.01)^x = 1500$$

$$1.01^x = 6$$

$$x \ln 1.01 = \ln 6$$

$$x = \frac{\ln 6}{\ln 1.01}$$

$$x \approx 180.070$$

55.  $2e^{5x} = 18$

$$e^{5x} = 9$$

$$5x = \ln 9$$

$$x = \frac{1}{5} \ln 9$$

$$x \approx 0.439$$

56.  $4e^{2x} = 40$

$$e^{2x} = 10$$

$$2x = \ln 10$$

$$x = \frac{1}{2} \ln 10 \approx 1.151$$

57.  $500e^{-x} = 300$

$$e^{-x} = \frac{3}{5}$$

$$-x = \ln \frac{3}{5}$$

$$x = -\ln \frac{3}{5} = \ln \frac{5}{3} \approx 0.511$$

58.  $1000e^{-4x} = 75$

$$e^{-4x} = \frac{3}{40}$$

$$\ln e^{-4x} = \ln \frac{3}{40}$$

$$-4x = \ln \frac{3}{40}$$

$$x = -\frac{1}{4} \ln \frac{3}{40} \approx 0.648$$

59.  $7 - 2e^x = 5$

$$-2e^x = -2$$

$$e^x = 1$$

$$x = \ln 1 = 0$$

60.  $-14 + 3e^x = 11$

$$3e^x = 25$$

$$e^x = \frac{25}{3}$$

$$\ln e^x = \ln \frac{25}{3}$$

$$x = \ln \frac{25}{3} \approx 2.120$$

61.  $e^{2x} - 4e^x - 5 = 0$

$$(e^x - 5)(e^x + 1) = 0$$

$$e^x = 5 \text{ or } e^x = -1$$

$$x = \ln 5 \approx 1.609$$

$$(e^x = -1 \text{ is impossible.})$$

62.  $e^{2x} - 5e^x + 6 = 0$

$$(e^x - 2)(e^x - 3) = 0$$

$$e^x = 2 \text{ or } e^x = 3$$

$$x = \ln 2 \approx 0.693 \text{ or}$$

$$x = \ln 3 \approx 1.099$$

63.  $250e^{0.02x} = 10,000$

$$e^{0.02x} = 40$$

$$0.02x = \ln 40$$

$$x = \frac{\ln 40}{0.02}$$

$$x \approx 184.444$$

64.  $100e^{0.005x} = 125,000$

$$e^{0.005x} = 1250$$

$$0.005x = \ln 1250$$

$$x = \frac{\ln 1250}{0.005}$$

$$x \approx 1426.180$$

65.  $e^x = e^{x^2-2}$

$$x = x^2 - 2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2, -1$$

66.  $e^{2x} = e^{x^2-8}$

$$2x = x^2 - 8$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x = 4, -2$$

67.  $e^{x^2-3x} = e^{x-2}$

$$x^2 - 3x = x - 2$$

$$x^2 - 4x + 2 = 0$$

$$x = \frac{4 \pm \sqrt{16-8}}{2}$$

$$x = 2 \pm \sqrt{2}$$

$$x \approx 3.414, 0.586$$

68.  $e^{-x^2} = e^{x^2-2x}$

$$-x^2 = x^2 - 2x$$

$$2x^2 - 2x = 0$$

$$2x(x - 1) = 0$$

$$x = 0, 1$$

69.  $\frac{400}{1 + e^{-x}} = 350$

$$1 + e^{-x} = \frac{400}{350} = \frac{8}{7}$$

$$e^{-x} = \frac{1}{7}$$

$$-x = \ln\left(\frac{1}{7}\right) = -\ln 7$$

$$x = \ln 7 \approx 1.946$$

70.  $\frac{525}{1 + e^{-x}} = 275$

$$1 + e^{-x} = \frac{525}{275}$$

$$e^{-x} = \frac{525}{275} - 1 = \frac{250}{275} = \frac{10}{11}$$

$$-x = \ln \frac{10}{11}$$

$$x = -\ln \frac{10}{11} = \ln \frac{11}{10} \approx 0.095$$

71.  $\frac{40}{1 - 5e^{-0.01x}} = 200$

$$1 - 5e^{-0.01x} = \frac{40}{200} = \frac{1}{5}$$

$$5e^{-0.01x} = \frac{4}{5}$$

$$e^{-0.01x} = \frac{4}{25}$$

$$-0.01x = \ln \left( \frac{4}{25} \right)$$

$$x = \frac{\ln(4/25)}{-0.01}$$

$$x \approx 183.258$$

72.  $\frac{50}{1 - 2e^{-0.001x}} = 1000$

$$1 - 2e^{-0.001x} = \frac{50}{1000} = 0.05$$

$$2e^{-0.001x} = 0.95$$

$$e^{-0.001x} = 0.475$$

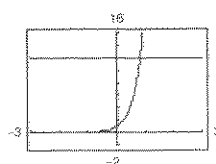
$$-0.001x = \ln 0.475$$

$$x = \frac{\ln 0.475}{-0.001}$$

$$x \approx 744.440$$

73.  $e^{3x} = 12$

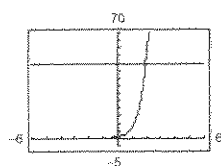
$x$	0.6	0.7	0.8	0.9	1.0
$f(x)$	6.05	8.17	11.02	14.88	20.09



$$x \approx 0.828$$

74.  $e^{2x} = 50$

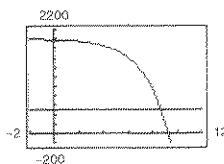
$x$	1.6	1.7	1.8	1.9	2.0
$e^{2x}$	24.53	29.96	36.60	44.70	54.60



$$x \approx 1.956$$

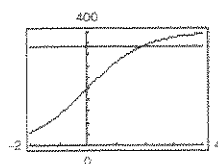
75.  $20(100 - e^{x/2}) = 500$

$x$	5	6	7	8	9
$f(x)$	1756	1598	1338	908	200



$$x \approx 8.635$$

76.  $\frac{400}{1 + e^{-x}} = 350$



$$x \approx 1.946$$

$x$	0	1	2	3	4
$\frac{400}{1 + e^{-x}}$	200	292	352	381	393

$$77. \left(1 + \frac{0.065}{365}\right)^{365t} = 4 \Rightarrow t = 21.330$$

$$78. \left(4 - \frac{2.471}{40}\right)^{9t} = 21$$

$$3.938225^{9t} = 21$$

The zero of  $y = 3.938225^{9t} - 21$  is  $t \approx 0.247$ .

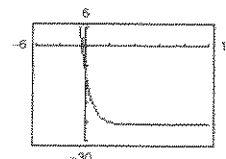
$$79. \frac{3000}{2 + e^{2x}} = 2$$

The zero of  $y = \frac{3000}{2 + e^{2x}} - 2$   
is  $x \approx 3.656$ .

$$80. \frac{119}{e^{6x} - 14} = 7$$

The zero of  $y = \frac{119}{e^{6x} - 14} - 7$   
is  $x \approx 0.572$ .

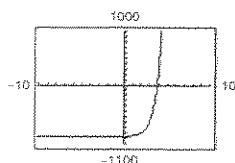
$$81. g(x) = 6e^{1-x} - 25$$



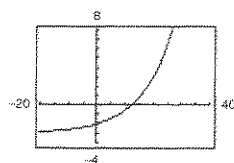
Zero at  $x \approx -0.427$

$$82. f(x) = 3e^{3x/2} - 962$$

The zero is  $x \approx 3.847$ .



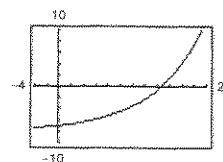
$$83. g(t) = e^{0.09t} - 3$$



Zero at  $t \approx 12.207$

$$84. h(t) = e^{0.125t} - 8$$

The zero is  $t \approx 16.636$ .



$$85. \ln x = -3$$

$$x = e^{-3} \approx 0.050$$

$$86. \ln x = -2$$

$$x = e^{-2} = \frac{1}{e^2} \approx 0.135$$

$$87. \ln 4x = 2.1$$

$$4x = e^{2.1}$$

$$x = \frac{1}{4}e^{2.1} \approx 2.042$$

$$88. \ln 2x = 1.5$$

$$e^{1.5} = 2x$$

$$x = \frac{1}{2}e^{1.5} \approx 2.241$$

$$89. -2 + 2 \ln 3x = 17$$

$$2 \ln 3x = 19$$

$$\ln 3x = \frac{19}{2}$$

$$3x = e^{19/2}$$

$$x = \frac{1}{3}e^{19/2}$$

$$x \approx 4453.242$$

$$90. 3 + 2 \ln x = 10$$

$$2 \ln x = 7$$

$$\ln x = \frac{7}{2} = 3.5$$

$$x = e^{3.5} \approx 33.115$$

$$91. \log_5(3x + 2) = \log_5(6 - x)$$

$$3x + 2 = 6 - x$$

$$4x = 4$$

$$x = 1$$

$$92. \log_9(4 + x) = \log_9(2x - 1)$$

$$4 + x = 2x - 1$$

$$x = 5$$

$$93. \log_{10}(z - 3) = 2$$

$$z - 3 = 10^2$$

$$z = 10^2 + 3$$

$$= 103$$



94.  $\log_{10} x^2 = 6$

$$x^2 = 10^6$$

$$x = \pm \sqrt{10^6} = \pm 1000$$

95.  $7 \log_4(0.6x) = 12$

$$\log_4(0.6x) = \frac{12}{7}$$

$$4^{12/7} = 0.6x = \frac{3}{5}x$$

$$x = \frac{5}{3} 4^{12/7}$$

$$\approx 17.945$$

96.  $4 \log_{10}(x - 6) = 11$

$$\log_{10}(x - 6) = \frac{11}{4}$$

$$10^{11/4} = x - 6$$

$$x = 6 + 10^{11/4}$$

$$\approx 568.341$$

97.  $\ln \sqrt{x+2} = 1$

$$\sqrt{x+2} = e^1$$

$$x+2 = e^2$$

$$x = e^2 - 2 \approx 5.389$$

98.  $\ln \sqrt{x-8} = 5$

$$\frac{1}{2} \ln(x-8) = 5$$

$$\ln(x-8) = 10$$

$$e^{10} = x - 8$$

$$x = 8 + e^{10}$$

$$\approx 22,034.466$$

99.  $\ln(x+1)^2 = 2$

$$e^{\ln(x+1)^2} = e^2$$

$$(x+1)^2 = e^2$$

$$x+1 = e \text{ or } x+1 = -e$$

$$x = e - 1 \approx 1.718$$

or

$$x = -e - 1 \approx -3.718$$

100.  $\ln(x^2 + 1) = 8$

$$e^8 = x^2 + 1$$

$$x = \pm \sqrt{e^8 - 1} \approx \pm 54.589$$

101.  $\log_4 x - \log_4(x-1) = \frac{1}{2}$

$$\log_4 \left( \frac{x}{x-1} \right) = \frac{1}{2}$$

$$4^{\log_4(x/(x-1))} = 4^{1/2}$$

$$\frac{x}{x-1} = 2$$

$$x = 2(x-1)$$

$$x = 2x - 2$$

$$2 = x$$

102.  $\log_3 x + \log_3(x-8) = 2$

$$\log_3[x(x-8)] = 2$$

$$3^2 = x(x-8) = x^2 - 8x$$

$$x^2 - 8x - 9 = 0$$

$$(x-9)(x+1) = 0$$

$$x = 9$$

( $x = -1$  is extraneous.)

103.  $\ln(x+5) = \ln(x-1) - \ln(x+1)$

$$\ln(x+5) = \ln \left( \frac{x-1}{x+1} \right)$$

$$x+5 = \frac{x-1}{x+1}$$

$$(x+5)(x+1) = x-1$$

$$x^2 + 6x + 5 = x - 1$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$$x = -2 \text{ or } x = -3$$

Both of these solutions are extraneous, so the equation has no solution.

104.  $\ln(x+1) - \ln(x-2) = \ln x, (x > 2)$

$$\ln\left(\frac{x+1}{x-2}\right) = \ln x$$

$$\frac{x+1}{x-2} = x$$

$$x+1 = x(x-2) = x^2 - 2x$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

Taking the positive solution,  $x = \frac{3 + \sqrt{13}}{2} \approx 3.303$ .

105.  $\log_{10} 8x - \log_{10}(1 + \sqrt{x}) = 2$

$$\log_{10} \frac{8x}{1 + \sqrt{x}} = 2$$

$$\frac{8x}{1 + \sqrt{x}} = 10^2$$

$$8x = 100 + 100\sqrt{x}$$

$$8x - 100\sqrt{x} - 100 = 0$$

$$2x - 25\sqrt{x} - 25 = 0$$

$$\sqrt{x} = \frac{25 \pm \sqrt{25^2 - 4(2)(-25)}}{4}$$

$$= \frac{25 \pm 5\sqrt{33}}{4}$$

Choosing the positive value, we have  $\sqrt{x} \approx 13.431$  and  $x \approx 180.384$ .

106.  $\log_{10} 4x - \log_{10}(12 + \sqrt{x}) = 2, (x > 0)$

$$\log_{10} \frac{4x}{12 + \sqrt{x}} = 2$$

$$\frac{4x}{12 + \sqrt{x}} = 10^2 = 100$$

$$4x = 1200 + 100\sqrt{x}$$

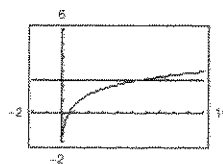
$$x - 25\sqrt{x} - 300 = 0, \text{ Quadratic in } \sqrt{x}$$

$$\sqrt{x} = \frac{25 \pm \sqrt{(-25)^2 - 4(-300)}}{2} = \frac{25 \pm \sqrt{1825}}{2}$$

Taking the positive root and squaring,  $x \approx 1146.5$ .

107.  $\ln 2x = 2.4$

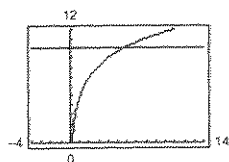
$x$	2	3	4	5	6
$f(x)$	1.39	1.79	2.08	2.30	2.48



$$x \approx 5.512$$

108.

$x$	4	5	6	7	8
$f(x)$	8.99	9.66	10.20	10.67	11.07



$$x \approx 5.606$$

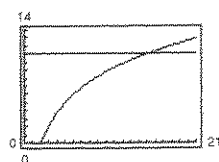
110.

$x$	150	155	160	165	170
$f(x)$	10.85	10.92	10.99	11.06	11.13

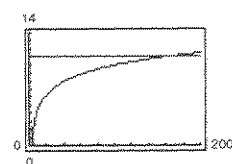
$$x \approx 160.489$$

109.  $6 \log_3(0.5x) = 11$

$x$	12	13	14	15	16
$f(x)$	9.79	10.22	10.63	11.00	11.36



$$x \approx 14.988$$



111.  $\log_{10} x = x^3 - 3$

Graphing  $y = \log_{10} x - x^3 + 3$ , you obtain two zeros,  $x \approx 1.469$  and  $x \approx 0.001$ .

112. Solving  $y = \log_{10} x^2 - 4 = 0$ ,  $x = \pm 100$ .

113.  $\ln x + \ln(x - 2) = 1$

Graphing  $y = \ln x + \ln(x - 2) - 1$ , you obtain one zero,  $x \approx 2.928$ .

114.  $\ln x + \ln(x + 1) = 2$

Graphing  $y = \ln x + \ln(x + 1) - 2$ , you obtain one zero,  $x \approx 2.264$ .

115.  $\ln(x - 3) + \ln(x + 3) = 1$

Graphing  $y = \ln(x - 3) + \ln(x + 3) - 1$ , you obtain  $x \approx 3.423$ .

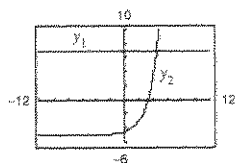
116.  $\ln x + \ln(x^2 + 4) = 10$

Solving  $y = \ln x + \ln(x^2 + 4) - 10 = 0$ ,  $x \approx 27.984$ .

117.  $y_1 = 7$

$$y_2 = 2^{x-1} - 5$$

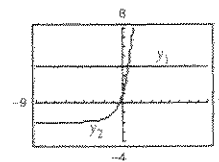
Intersection: (4.585, 7)



118.  $y_1 = 4$

$$y_2 = 3^{x+1} - 2$$

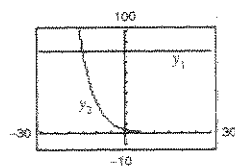
The graphs intersect at  $(x, y) \approx (0.631, 4)$ .



119.  $y_1 = 80$

$$y_2 = 4e^{-0.2x}$$

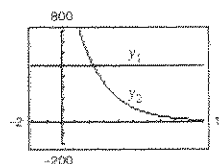
Intersection: (-14.979, 80)



120.  $y_1 = 500$

$$y_2 = 1500e^{-x/2}$$

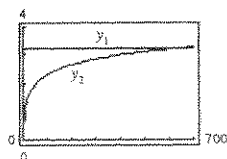
From the graph, we have  $(x, y) \approx (2.197, 500)$ .



121.  $y_1 = 3.25$

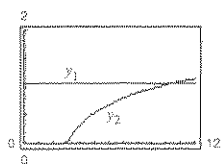
$$y_2 = \frac{1}{2} \ln(x + 2)$$

Intersection: (663.142, 3.25)



122.  $y_1 = 1.05$

$$y_2 = \ln \sqrt{x-2} = \frac{1}{2} \ln(x-2)$$



The graphs intersect at (10.166, 1.05).

124.  $-x^2e^{-x} + 2xe^{-x} = 0$

$$(-x^2 + 2x)e^{-x} = 0$$

$$-x^2 + 2x = 0 \quad (\text{since } e^{-x} \neq 0)$$

$$-x(x-2) = 0$$

$$x = 0, 2$$

126.  $e^{-2x} - 2xe^{-2x} = 0$

$$(1 - 2x)e^{-2x} = 0$$

$$1 - 2x = 0 \quad (\text{since } e^{-2x} \neq 0)$$

$$x = \frac{1}{2}$$

128.  $\frac{1 - \ln x}{x^2} = 0$

$$1 - \ln x = 0 \quad (x > 0)$$

$$\ln x = 1$$

$$x = e \approx 2.718$$

130.  $2x \ln\left(\frac{1}{x}\right) - x = 0$

$$x\left(2 \ln\left(\frac{1}{x}\right) - 1\right) = 0$$

$$2 \ln\left(\frac{1}{x}\right) - 1 = 0 \quad (\text{since } x > 0)$$

$$\ln\left(\frac{1}{x}\right) = \frac{1}{2}$$

$$\frac{1}{x} = e^{1/2}$$

$$x = e^{-1/2} \approx 0.607$$

123.  $2x^2e^{2x} + 2xe^{2x} = 0$

$$(2x^2 + 2x)e^{2x} = 0$$

$$2x^2 + 2x = 0 \quad (\text{since } e^{2x} \neq 0)$$

$$2x(x+1) = 0$$

$$x = 0, -1$$

125.  $-xe^{-x} + e^{-x} = 0$

$$(-x + 1)e^{-x} = 0$$

$$-x + 1 = 0 \quad (\text{since } e^{-x} \neq 0)$$

$$x = 1$$

127.  $2x \ln x + x = 0$

$$x(2 \ln x + 1) = 0$$

$$2 \ln x + 1 = 0 \quad (\text{since } x > 0)$$

$$\ln x = -\frac{1}{2}$$

$$x = e^{-1/2} \approx 0.607$$

129.  $\frac{1 + \ln x}{2} = 0$

$$1 + \ln x = 0$$

$$\ln x = -1$$

$$x = e^{-1} = \frac{1}{e} \approx 0.368$$

131. (a)  $2000 = 1000e^{0.075t}$

$$2 = e^{0.075t}$$

$$\ln 2 = 0.075t$$

$$t = \frac{\ln 2}{0.075} \approx 9.24 \text{ years}$$

(b)  $3000 = 1000e^{0.075t}$

$$3 = e^{0.075t}$$

$$\ln 3 = 0.075t$$

$$t = \frac{\ln 3}{0.075} \approx 14.65 \text{ years}$$

132. (a)  $2000 = 1000e^{0.06t}$

$$2 = e^{0.06t}$$

$$\ln 2 = 0.06t$$

$$t = \frac{\ln 2}{0.06} \approx 11.55 \text{ years}$$

(b)  $3000 = 1000e^{0.06t}$

$$3 = e^{0.06t}$$

$$\ln 3 = 0.06t$$

$$t = \frac{\ln 3}{0.06} \approx 18.31 \text{ years}$$

134. (a)  $2000 = 1000e^{0.0375t}$

$$2 = e^{0.0375t}$$

$$\ln 2 = 0.0375t$$

$$t = \frac{\ln 2}{0.0375} \approx 18.48 \text{ years}$$

(b)  $3000 = 1000e^{0.0375t}$

$$3 = e^{0.0375t}$$

$$\ln 3 = 0.0375t$$

$$t = \frac{\ln 3}{0.0375} \approx 29.30 \text{ years}$$

136.  $p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$

(a) When  $p = \$600$ :

$$600 = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$0.12 = 1 - \frac{4}{4 + e^{-0.002x}}$$

$$\frac{4}{4 + e^{-0.002x}} = 0.88$$

$$4 = 3.52 + 0.88e^{-0.002x}$$

$$0.48 = 0.88e^{-0.002x}$$

$$\frac{6}{11} = e^{-0.002x}$$

$$\ln \frac{6}{11} = \ln e^{-0.002x}$$

$$\ln \frac{6}{11} = -0.002x$$

$$x = \frac{\ln(6/11)}{-0.002} \approx 303 \text{ units}$$

133. (a)  $2000 = 1000e^{0.025t}$

$$2 = e^{0.025t}$$

$$\ln 2 = 0.025t$$

$$t = \frac{\ln 2}{0.025} \approx 27.73 \text{ years}$$

(b)  $3000 = 1000e^{0.025t}$

$$3 = e^{0.025t}$$

$$\ln 3 = 0.025t$$

$$t = \frac{\ln 3}{0.025} \approx 43.94 \text{ years}$$

135.  $p = 500 - 0.5(e^{0.004x})$

(a)  $p = 350$

$$350 = 500 - 0.5(e^{0.004x})$$

$$300 = e^{0.004x}$$

$$0.004x = \ln 300$$

$$x \approx 1426 \text{ units}$$

(b)  $p = 300$

$$300 = 500 - 0.5(e^{0.004x})$$

$$400 = e^{0.004x}$$

$$0.004x = \ln 400$$

$$x \approx 1498 \text{ units}$$

(b) When  $p = \$400$ :

$$400 = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$0.08 = 1 - \frac{4}{4 + e^{-0.002x}}$$

$$\frac{4}{4 + e^{-0.002x}} = 0.92$$

$$4 = 3.68 + 0.92e^{-0.002x}$$

$$0.32 = 0.92e^{-0.002x}$$

$$\frac{8}{23} = e^{-0.002x}$$

$$\ln \frac{8}{23} = \ln e^{-0.002x}$$

$$x = \frac{\ln(8/23)}{-0.002} \approx 528 \text{ units}$$

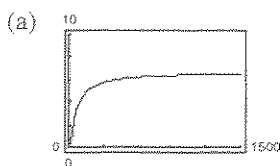
137.  $7247 - 596.5 \ln t = 5800$

$$-596.5 \ln t = -1447$$

$$\ln t \approx 2.4258$$

$$t \approx 11.3, \text{ or } 2001$$

138.  $V = 6.7e^{-48.1/t}, t > 0$

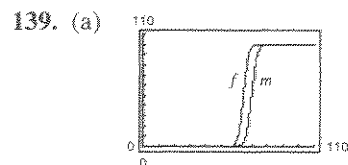
(b) As  $t \rightarrow \infty, V \rightarrow 6.7$ .Horizontal asymptote:  $y = 6.7$ The yield will approach  
6.7 million cubic feet per acre.

(c)  $1.3 = 6.7e^{-48.1/t}$

$$\frac{1.3}{6.7} = e^{-48.1/t}$$

$$\ln \frac{1.3}{6.7} = \frac{-48.1}{t}$$

$$t = \frac{-48.1}{\ln(1.3/6.7)} \approx 29.3 \text{ years}$$

(b) From the graph we see horizontal asymptotes at  $y = 0$  and  $y = 100$ . These represent the lower and upper percent bounds.

(c) Males:  $50 = \frac{100}{1 + e^{-0.6114(x-69.71)}}$

$$1 + e^{-0.6114(x-69.71)} = 2$$

$$e^{-0.6114(x-69.71)} = 1$$

$$-0.6114(x - 69.71) = \ln 1$$

$$-0.6114(x - 69.71) = 0$$

$$x = 69.71 \text{ inches}$$

Females:  $50 = \frac{100}{1 + e^{-0.66607(x-64.51)}}$

$$1 + e^{-0.66607(x-64.51)} = 2$$

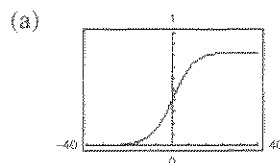
$$e^{-0.66607(x-64.51)} = 1$$

$$-0.66607(x - 64.51) = \ln 1$$

$$-0.66607(x - 64.51) = 0$$

$$x = 64.51 \text{ inches}$$

140.  $P = \frac{0.83}{1 + e^{-0.2n}}$

(b) Horizontal asymptotes:  $y = 0, y = 0.83$ The upper asymptote,  $y = 0.83$ , indicates that the proportion of correct responses will approach 0.83 as the number of trials increases.(c) When  $P = 60\%$  or  $P = 0.60$ :

$$0.60 = \frac{0.83}{1 + e^{-0.2n}}$$

$$1 + e^{-0.2n} = \frac{0.83}{0.60}$$

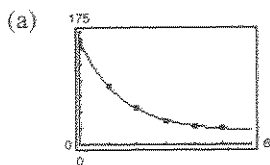
$$e^{-0.2n} = \frac{0.83}{0.60} - 1$$

$$\ln e^{-0.2n} = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$-0.2n = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$n = -\frac{\ln\left(\frac{0.83}{0.60} - 1\right)}{0.2} \approx 5 \text{ trials}$$

141.  $T = 20[1 + 7(2^{-h})]$



(b) We see a horizontal asymptote at  $y = 20$ .  
This represents the room temperature.

(c)  $100 = 20[1 + 7(2^{-h})]$

$$5 = 1 + 7(2^{-h})$$

$$4 = 7(2^{-h})$$

$$\frac{4}{7} = 2^{-h}$$

$$\ln\left(\frac{4}{7}\right) = \ln 2^{-h}$$

$$\ln\left(\frac{4}{7}\right) = -h \ln 2$$

$$\frac{\ln(4/7)}{-\ln 2} = h$$

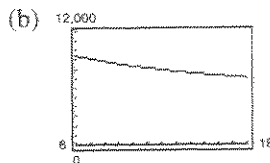
$$h \approx 0.81 \text{ hour}$$

142. (a)  $13,387 - 2190.5 \ln t = 7250$

$$2190.5 \ln t = 6137$$

$$\ln t = 2.8016$$

$$t \approx 16.5, \text{ or } 2006$$



(c) Let  $y_1 = 13,387 - 2190.5 \ln t$  and  $y_2 = 7250$ .  
The graphs of  $y_1$  and  $y_2$  intersect at  $t \approx 16.5$ .

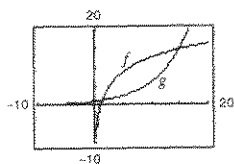
 143. False. The equation  $e^x = 0$  has no solutions.

 144. False. A logarithmic equation can have any number of extraneous solutions. For example  $\ln(2x - 1) + \ln(x + 2) = \ln(x^2 - x - 5)$  has two extraneous solutions,  $x = -1$  and  $x = -3$ .

145. Answers will vary.

146.  $f(x) = \log_a x$ ,  $g(x) = a^x$ ,  $a > 1$ .

(a)  $a = 1.2$



The curves intersect twice: (1.258, 1.258) and (14.767, 14.767)

(b) If  $f(x) = \log_a x = a^x = g(x)$  intersect exactly once, then

$$x = \log_a x = a^x \Rightarrow a = x^{1/x}.$$

The graphs of  $y = x^{1/x}$  and  $y = a$  intersect once for  $a = e^{1/e} \approx 1.445$ . Then

$$\log_a x = x \Rightarrow (e^{1/e})^x = x \Rightarrow e^{x/e} = x \Rightarrow x = e.$$

For  $a = e^{1/e}$ , the curves intersect once at  $(e, e)$ .

(c) For  $1 < a < e^{1/e}$  the curves intersect twice. For  $a > e^{1/e}$ , the curves do not intersect.

147. Yes. The doubling time is given by

$$2P = Pe^{rt}$$

$$2 = e^{rt}$$

$$\ln 2 = rt$$

$$t = \frac{\ln 2}{r}$$

The time to quadruple is given by

$$4P = Pe^{rt}$$

$$4 = e^{rt}$$

$$\ln 4 = rt$$

$$t = \frac{\ln 4}{r} = \frac{\ln 2^2}{r} = \frac{2 \ln 2}{r} = 2 \left[ \frac{\ln 2}{r} \right]$$

which is twice as long.

148. To find the length of time it takes for an investment  $P$  to double to  $2P$ , solve

$$2P = Pe^{rt}$$

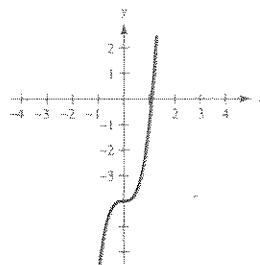
$$2 = e^{rt}$$

$$\ln 2 = rt$$

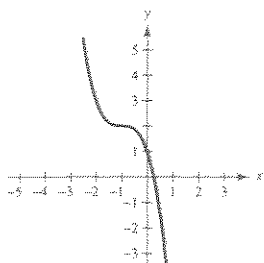
$$\frac{\ln 2}{r} = t.$$

Thus, you can see that the time is not dependent on the size of the investment, but rather the interest rate.

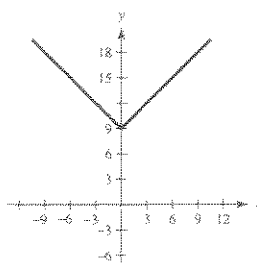
149.  $f(x) = 3x^3 - 4$



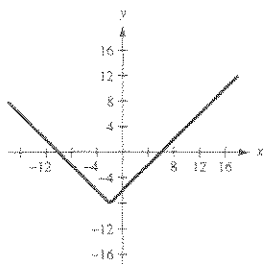
150.  $f(x) = -(x+1)^3 + 2$



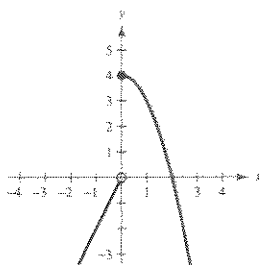
151.  $f(x) = |x| + 9$



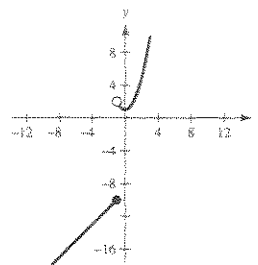
152.  $f(x) = |x+2| - 8$



153.  $f(x) = \begin{cases} 2x, & x < 0 \\ -x^2 + 4, & x \geq 0 \end{cases}$



154.  $f(x) = \begin{cases} x - 9, & x \leq -1 \\ x^2 + 1, & x > -1 \end{cases}$





## Section 3.5 Exponential and Logarithmic Models

- You should be able to solve compound interest problems.

$$1. A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2. A = Pe^{rt}$$

- You should be able to solve growth and decay problems.

(a) Exponential growth if  $b > 0$  and  $y = ae^{bx}$ .

(b) Exponential decay if  $b > 0$  and  $y = ae^{-bx}$ .

- You should be able to use the Gaussian model

$$y = ae^{-(x-b)^2/c}$$

- You should be able to use the logistics growth model

$$y = \frac{a}{1 + be^{-(x-c)/d}}$$

- You should be able to use the logarithmic models

$$y = \ln(ax + b) \text{ and } y = \log_{10}(ax + b).$$

## Vocabulary Check

1. (a) iv (b) i (c) vi (d) iii (e) vii (f) ii (g) v

2. Normally

3. Sigmoidal

4. Bell-shaped, mean

1.  $y = 2e^{x/4}$

This is an exponential growth model.

Matches graph (c).

2.  $y = 6e^{-x/4}$

This is an exponential decay model.

Matches graph (e).

3.  $y = 6 + \log_{10}(x + 2)$

This is a logarithmic model, and contains  $(-1, 6)$ .

Matches graph (b).

4.  $y = 3e^{-(x-2)^2/5}$

Gaussian model

Matches (a).

5.  $y = \ln(x + 1)$

This is a logarithmic model.

Matches graph (d).

6.  $y = \frac{4}{1 + e^{-2x}}$

Logistics model

Matches (f).

7. Since  $A = 10,000e^{0.035t}$ , the time to double is given by

$$20,000 = 10,000e^{0.035t}$$

$$2 = e^{0.035t}$$

$$\ln 2 = 0.035t$$

$$t = \frac{\ln 2}{0.035} \approx 19.8 \text{ years.}$$

Amount after 10 years:

$$A = 10,000e^{0.035(10)} \approx \$14,190.68$$

8. Since  $A = 2000e^{0.015t}$ , the time to double is given by

$$4000 = 2000e^{0.015t}$$

$$2 = e^{0.015t}$$

$$\ln 2 = 0.015t$$

$$t = \frac{\ln 2}{0.015} \approx 46.2 \text{ years.}$$

Amount after 10 years:

$$A = 2000e^{0.015(10)} \approx \$2323.67$$

9. Since  $A = 7500e^{rt}$  and  $A = 15,000$  when  $t = 21$ , we have the following.

$$15,000 = 7500e^{21r}$$

$$2 = e^{21r}$$

$$\ln 2 = 21r$$

$$r = \frac{\ln 2}{21} \approx 0.033 = 3.3\%$$

Amount after 10 years:

$$A = 7500e^{0.033(10)} \approx \$10,432.26$$

11. Since  $A = 5000e^{rt}$  and  $A = 5665.74$  when  $t = 10$ , we have the following.

$$5665.74 = 5000e^{10r}$$

$$\frac{5665.74}{5000} = e^{10r}$$

$$\ln\left(\frac{5665.74}{5000}\right) = 10r$$

$$r = \frac{1}{10} \ln\left(\frac{5665.74}{5000}\right) \\ \approx 0.0125 = 1.25\%$$

The time to double is given by

$$10,000 = 5000e^{0.0125t}$$

$$2 = e^{0.0125t}$$

$$\ln 2 = 0.0125t$$

$$t = \frac{\ln 2}{0.0125} \approx 55.5 \text{ years.}$$

13. Since  $A = Pe^{0.045t}$  and  $A = 100,000$  when  $t = 10$ , we have the following.

$$100,000 = Pe^{0.045(10)}$$

$$\frac{100,000}{e^{0.45}} = P \approx 63,762.82$$

The time to double is given by

$$127,525.64 = 63,762.82e^{0.045t}$$

$$2 = e^{0.045t}$$

$$\ln 2 = 0.045t$$

$$t = \frac{\ln 2}{0.045} \approx 15.4 \text{ years.}$$

10. Since  $A = 1000e^{rt}$  and  $A = 2000$  when  $t = 12$ , we have the following.

$$2000 = 1000e^{12r}$$

$$2 = e^{12r}$$

$$\ln 2 = 12r$$

$$r = \frac{\ln 2}{12} \approx 0.058 = 5.8\%$$

Amount after 10 years:

$$A = 1000e^{0.058(10)} \approx \$1786.04$$

12. Since  $A = 300e^{rt}$  and  $A = 385.21$  when  $t = 10$ , we have the following.

$$385.21 = 300e^{10r}$$

$$\frac{385.21}{300} = e^{10r}$$

$$\ln\left(\frac{385.21}{300}\right) = 10r$$

$$r = \frac{1}{10} \ln\left(\frac{385.21}{300}\right) \\ \approx 0.025 = 2.5\%$$

The time to double is given by

$$600 = 300e^{0.025t}$$

$$2 = e^{0.025t}$$

$$\ln 2 = 0.025t$$

$$t = \frac{\ln 2}{0.025} \approx 27.7 \text{ years.}$$

14. Since  $A = Pe^{0.02t}$  and  $A = 2500$  when  $t = 10$ , we have the following.

$$2500 = Pe^{0.02(10)}$$

$$\frac{2500}{e^{0.02}} = P \approx \$2046.83$$

The time to double is given by

$$4093.66 = 2046.83e^{0.02t}$$

$$2 = e^{0.02t}$$

$$\ln 2 = 0.02t$$

$$t = \frac{\ln 2}{0.02} \approx 34.7 \text{ years.}$$

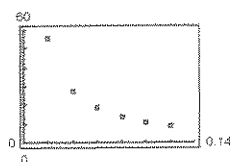
15.  $3P = Pe^{rt}$

$3 = e^{rt}$

$\ln 3 = rt$

$\frac{\ln 3}{r} = t$

$r$	2%	4%	6%	8%	10%	12%
$t = \frac{\ln 3}{r}$	54.93	27.47	18.31	13.73	10.99	9.16



16.  $3P = P(1 + r)^t$

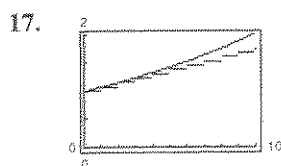
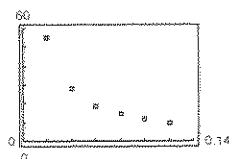
$3 = (1 + r)^t$

$\ln 3 = \ln(1 + r)^t$

$\ln 3 = t \ln(1 + r)$

$\frac{\ln 3}{\ln(1 + r)} = t$

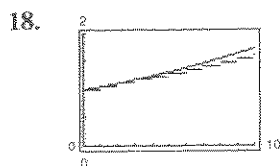
$r$	2%	4%	6%	8%	10%	12%
$t = \frac{\ln 3}{\ln(1 + r)}$	55.48	28.01	18.85	14.27	11.53	9.69



Continuous compounding results in faster growth.

$A = 1 + 0.075[t]$

and  $A = e^{0.07t}$



$A = 1 + 0.06[t]$

$A = \left(1 + \frac{0.055}{365}\right)^{[365t]}$

From the graph,  $5\frac{1}{2}\%$  compounded daily grows faster than 6% simple interest.

19.  $\frac{1}{2}C = Ce^{k(1599)}$

$\frac{1}{2} = e^{1599k}$

$k = \frac{\ln(1/2)}{1599}$

$y = Ce^{kt}$

$= 10e^{[\ln(1/2)/1599]1000}$

$\approx 6.48 \text{ g}$

20.  $\frac{1}{2}C = Ce^{k(1599)}$

$\frac{1}{2} = e^{1599k}$

$k = \frac{\ln(1/2)}{1599}$

$y = Ce^{kt}$

$1.5 = Ce^{[\ln(1/2)/1599]1000}$

$1.5 \approx C(0.64824)$

$C \approx 2.31 \text{ g}$

21.  $\frac{1}{2}C = Ce^{k(5715)}$

$\frac{1}{2} = e^{5715k}$

$k = \frac{\ln(1/2)}{5715}$

$y = Ce^{kt}$

$= 3e^{[\ln(1/2)/5715]1000}$

$\approx 2.66 \text{ g}$

$$22. \frac{1}{2}C = Ce^{k(24,100)}$$

$$\frac{1}{2} = e^{24,100k}$$

$$k = \frac{\ln(1/2)}{24,100}$$

$$0.4 = Ce^{[\ln(1/2)/24,100]1000}$$

$$\approx C(0.97165)$$

$$C \approx 0.41 \text{ g}$$

$$23. y = ae^{bx}$$

$$1 = ae^{b(0)} \Rightarrow 1 = a$$

$$10 = e^{b(3)}$$

$$\ln 10 = 3b$$

$$\frac{\ln 10}{3} = b \Rightarrow b \approx 0.7675$$

$$\text{Thus, } y = e^{0.7675x}.$$

$$24. y = ae^{bx}$$

$$\frac{1}{2} = ae^{b(0)} \Rightarrow a = \frac{1}{2}$$

$$5 = \frac{1}{2}e^{b(4)}$$

$$10 = e^{4b}$$

$$\ln 10 = 4b$$

$$b = \frac{\ln 10}{4} \approx 0.5756$$

$$\text{Thus, } y = \frac{1}{2}e^{0.5756x}.$$

$$25. (0, 4) \Rightarrow a = 4$$

$$(5, 1) \Rightarrow 1 = 4e^{b(5)} \Rightarrow b = \frac{1}{5} \ln\left(\frac{1}{4}\right)$$

$$= -\frac{1}{5} \ln 4 \approx -0.2773$$

$$y = 4e^{-0.2773x}$$

$$26. y = ae^{bx}$$

$$1 = ae^{b(0)} \Rightarrow 1 = a$$

$$\frac{1}{4} = e^{b(3)}$$

$$\ln\left(\frac{1}{4}\right) = 3b$$

$$\frac{\ln(1/4)}{3} = b \Rightarrow b \approx -0.4621$$

$$\text{Thus, } y = e^{-0.4621x}.$$

$$27. (a) \text{ Australia: } (0, 19.2), (10, 20.9)$$

$$a = 19.2 \text{ and } 20.9 = 19.2e^{b(10)} \Rightarrow b = 0.008484$$

$$y = 19.2e^{0.008484x}$$

$$\text{For 2030, } y \approx 24.8 \text{ million.}$$

$$\text{Canada: } (0, 31.3), (10, 34.3)$$

$$a = 31.3 \text{ and } 34.3 = 31.3e^{b(10)} \Rightarrow b = 0.009153$$

$$y = 31.3e^{0.009153x}$$

$$\text{For 2030, } y \approx 41.2 \text{ million.}$$

$$\text{Philippines: } (0, 79.7), (10, 95.9)$$

$$a = 79.7 \text{ and } 95.9 = 79.7e^{b(10)} \Rightarrow b = 0.0185$$

$$y = 79.7e^{0.0185x}$$

$$\text{For 2030, } y \approx 138.8 \text{ million.}$$

$$\text{South Africa: } (0, 44.1), (10, 43.3)$$

$$a = 44.1 \text{ and } 43.3 = 44.1e^{b(10)} \Rightarrow b = -0.00183$$

$$y = 44.1e^{-0.00183x}$$

$$\text{For 2030, } y \approx 41.7 \text{ million.}$$

—CONTINUED—

## 27. (a) —CONTINUED—

 Turkey:  $(0, 65.7), (10, 73.3)$ 

$$a = 65.7 \text{ and } 73.3 = 65.7e^{b(10)} \Rightarrow b = 0.01095$$

$$y = 65.7e^{0.01095t}$$

 For 2030,  $y \approx 91.2$  million.

 (b) The constant  $b$  gives the growth rates.

 (c) The constant  $b$  is negative for South Africa.

28.  $P = 372.55e^{-0.01052t}$

(a) Decreasing because the exponent is negative.

 (b) For 1990,  $t = 0$  and  $P \approx 372,550$  people.

 For 2000,  $t = 10$  and  $P \approx 335,349$  people.

 For 2004,  $t = 14$  and  $P \approx 321,530$  people.

 (c)  $300 = 372.55e^{-0.01052t} \Rightarrow t \approx 20.6$ , or 2010

29. (a)  $180 = 134.0e^{k(10)}$

$$10k = \ln \frac{180}{134.0}$$

$$k \approx 0.0295$$

 (b) For 2010,  $t = 20$  and

$$P = 134.0e^{0.0295(20)} \approx 241,734 \text{ people.}$$

30.  $P = 258.0e^{kt}$

 (a)  $478 = 258.0e^{k(10)}$ 

$$10k = \ln \left( \frac{478}{258} \right)$$

$$k \approx 0.0617$$

 (b) For 2010,  $t = 20$  and

$$P = 258e^{0.0617(20)} \approx 886,215 \text{ people.}$$

31.  $y = Ce^{kt}$

$$\frac{1}{2}C = Ce^{(1599)k}$$

$$\ln \left( \frac{1}{2} \right) = 1599k$$

$$k = \frac{\ln(1/2)}{1599}$$

 When  $t = 100$ , we have

$$y = Ce^{[(\ln(1/2) \cdot 100)/1599]} \approx 0.958C, \text{ or } 95.8\%.$$

32.  $y = Ce^{kt}$

$$\frac{1}{2}C = Ce^{5715k}$$

$$\ln \left( \frac{1}{2} \right) = 5715k$$

$$k = \frac{\ln(1/2)}{5715}$$

The ancient charcoal has only 15% as much radioactive carbon.

$$0.15C = Ce^{[(\ln(1/2)/5715)t]}$$

$$\ln 0.15 = \frac{\ln(1/2)}{5715}t$$

$$t \approx 15,642 \text{ years}$$

33. (a)  $V = mt + b, V(0) = 30,788 \Rightarrow b = 30,788$

$$V(2) = 24,000 \Rightarrow 24,000 = 2m + 30,788$$

$$\Rightarrow m = -3394$$

$$V(t) = -3394t + 30,788$$

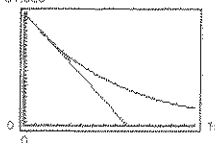
(b)  $V = ae^{kt}, V(0) = 30,788 \Rightarrow b = 30,788$

$$V(2) = 24,000 \Rightarrow 24,000 = 30,788e^{2k}$$

$$\Rightarrow k = \frac{1}{2} \ln \left( \frac{24,000}{30,788} \right) \approx -0.1245$$

$$V = 30,788e^{-0.1245t}$$

(c)



(d) The exponential model depreciates faster in the first year.

(e) Answers will vary.

34. Let  $t = 0$  correspond to 2005.

(a)  $V = mt + b$ ,  $V(0) = 1150 \Rightarrow b = 1150$

$V(2) = 550 \Rightarrow 550 = m(2) + 1150 \Rightarrow m = -300$

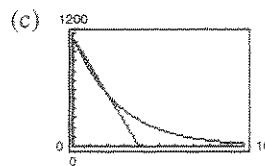
$V(t) = -300t + 1150$

(b)  $V = ae^{kt}$ ,  $V(0) = 1150 \Rightarrow a = 1150$

$V(2) = 550 \Rightarrow 550 = 1150e^{2k}$

$\Rightarrow k = \frac{1}{2} \ln\left(\frac{55}{115}\right) \approx -0.3688$

$V(t) = 1150e^{-0.3688t}$



(d) The exponential model depreciates faster in the first year.

(e) Answers will vary.

35.  $S(t) = 100(1 - e^{kt})$

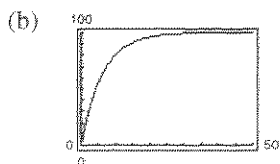
(a)  $15 = 100(1 - e^{k(1)})$

$-85 = -100e^k$

$k = \ln 0.85$

$k \approx -0.1625$

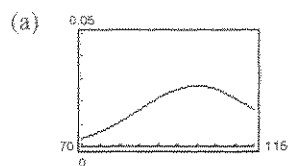
$S(t) = 100(1 - e^{-0.1625t})$



(c)  $S(5) = 100(1 - e^{-0.1625(5)})$

$\approx 55.625 = 55,625$  units

37.  $y = 0.0266e^{-(x-100)^2/450}$ ,  $70 \leq x \leq 115$



(b) Maximum point is  $x = 100$ , the average IQ score.

36.  $S = 10(1 - e^{kx})$

$x = 5$  (in hundreds)

$S = 2.5$  (in thousands)

(a)  $2.5 = 10(1 - e^{k(5)})$

$0.25 = 1 - e^{5k}$

$e^{5k} = 0.75$

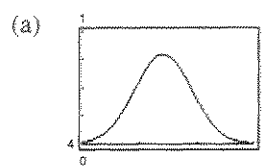
$5k = \ln 0.75$

$k \approx -0.0575$

$S = 10(1 - e^{-0.0575x})$

(b) When  $x = 7$ ,  $S = 10(1 - e^{-0.0575(7)}) \approx 3.314$  which corresponds to 3314 units.

38.  $y = 0.7979e^{-(x-5.4)^2/0.5}$



(b) About 5.4 hours

$$39. p(t) = \frac{1000}{1 + 9e^{-0.1656t}}$$

$$(a) p(5) = \frac{1000}{1 + 9e^{-0.1656(5)}} \approx 203 \text{ animals}$$

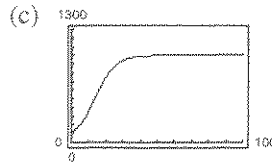
$$(b) 500 = \frac{1000}{1 + 9e^{-0.1656t}}$$

$$1 + 9e^{-0.1656t} = 2$$

$$9e^{-0.1656t} = 1$$

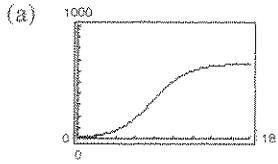
$$e^{-0.1656t} = \frac{1}{9}$$

$$t = \frac{-\ln(1/9)}{0.1656} \approx 13 \text{ months}$$



The horizontal asymptotes are  $p = 0$  and  $p = 1000$ . The population will approach 1000 as time increases.

$$40. y = \frac{663}{1 + 72e^{-0.547t}}, 0 \leq t \leq 18$$



(b) For  $t = 19$ ,  $y \approx 662$ .

For  $t = 30$ ,  $y \approx 663$ .

(c) As  $t \rightarrow \infty$ ,  $y \rightarrow \frac{663}{1} = 663$ , limiting value.

(d) Answers will vary.

$$41. R = \log_{10}\left(\frac{I}{I_0}\right) = \log_{10}(I) \Rightarrow I = 10^R$$

(a)  $I = 10^{6.1} \approx 1,258,925$

(b)  $I = 10^{7.6} \approx 39,810,717$

(c)  $I = 10^{9.0} \approx 1,000,000,000$

$$42. R = \log_{10}\left(\frac{I}{I_0}\right) = \log_{10}(I)$$

(a)  $R = \log_{10}(39,811,000) \approx 7.6$

(b)  $R = \log_{10}(12,589,000) \approx 7.1$

(c)  $R = \log_{10}(251,200) \approx 5.4$

$$43. \beta(I) = 10 \log_{10}(I/I_0), \text{ where } I_0 = 10^{-12} \text{ watt per square meter.}$$

(a)  $\beta(10^{-10}) = 10 \cdot \log_{10}\left(\frac{10^{-10}}{10^{-12}}\right) = 10 \log_{10} 10^2 = 20 \text{ decibels}$

(b)  $\beta(10^{-5}) = 10 \cdot \log_{10}\left(\frac{10^{-5}}{10^{-12}}\right) = 10 \log_{10} 10^7 = 70 \text{ decibels}$

(c)  $\beta(10^0) = 10 \cdot \log_{10}\left(\frac{10^0}{10^{-12}}\right) = 10 \log_{10} 10^{12} = 120 \text{ decibels}$

$$44. \beta = 10 \log_{10}\left(\frac{I}{I_0}\right) = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$$

(a)  $\beta(10^{-4}) = 10 \log_{10}\left(\frac{10^{-4}}{10^{-12}}\right) = 10 \log_{10}(10^8) = 80 \text{ decibels}$

(b)  $\beta(10^{-3}) = 90 \text{ decibels}$

(c)  $\beta(10^{-2}) = 100 \text{ decibels}$

$$45. \quad \beta = 10 \log_{10} \left( \frac{I}{I_0} \right)$$

$$10^{\beta/10} = \frac{I}{I_0}$$

$$I = I_0 10^{\beta/10}$$

$$\begin{aligned} \% \text{ decrease} &= \frac{I_0 10^{8.8} - I_0 10^{7.2}}{I_0 10^{8.8}} \times 100 \\ &= 97.5\% \end{aligned}$$

$$47. \quad \text{pH} = -\log_{10} [\text{H}^+] = -\log_{10} [2.3 \times 10^{-5}] \approx 4.64$$

$$49. \quad \text{pH} = -\log_{10} [\text{H}^+]$$

$$-\text{pH} = \log_{10} [\text{H}^+]$$

$$10^{-\text{pH}} = [\text{H}^+]$$

$$\frac{\text{Hydrogen ion concentration of grape}}{\text{Hydrogen ion concentration of milk of magnesia}}$$

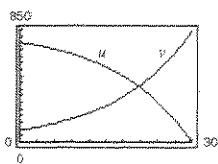
$$= \frac{10^{-3.5}}{10^{-10.5}} = 10^7$$

$$51. (a) \quad P = 120,000, r = 0.075, M = 839.06$$

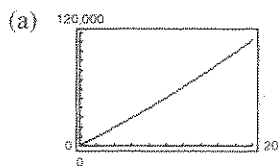
$$u = M - \left( M - \frac{Pr}{12} \right) \left( 1 + \frac{r}{12} \right)^{12t}$$

$$= 839.06 - (839.06 - 750)(1 + 0.00625)^{12t}$$

$$v = (839.06 - 750)(1.00625)^{12t}$$



$$52. \quad u = 120,000 \left[ \frac{0.075t}{1 - \left( \frac{1}{1 + 0.075/12} \right)^{12t}} - 1 \right]$$



(b) From the graph, when  $u = 120,000$ ,  $t \approx 21.2$  years. Yes, a mortgage of approximately 37.6 years will result in about \$240,000 of interest.

$$46. \quad \beta = 10 \log_{10} \left( \frac{I}{I_0} \right)$$

$$10^{\beta/10} = \frac{I}{I_0}$$

$$I = I_0 10^{\beta/10}$$

$$\% \text{ decrease} = \frac{I_0 10^{9.3} - I_0 10^{8.0}}{I_0 10^{9.3}} \times 100 \approx 95\%$$

$$48. \quad 5.8 = -\log_{10} [\text{H}^+]$$

$$10^{-5.8} = [\text{H}^+]$$

$$[\text{H}^+] \approx 1.58 \times 10^{-6} \text{ moles per liter}$$

$$50. \quad \text{pH} - 1 = -\log_{10} [\text{H}^+]$$

$$-(\text{pH} - 1) = \log_{10} [\text{H}^+]$$

$$10^{-(\text{pH} - 1)} = [\text{H}^+]$$

$$10^{-\text{pH} + 1} = [\text{H}^+]$$

$$10^{-\text{pH}} \cdot 10 = [\text{H}^+]$$

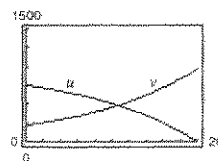
The hydrogen ion concentration is increased by a factor of 10.

(b) In the early years, the majority of the monthly payment goes toward interest. The interest and principle are equal when  $t \approx 20.729 \approx 21$  years.

$$(c) \quad P = 120,000, r = 0.075, M = 966.71$$

$$u = 966.71 - (966.71 - 750)(1.00625)^{12t}$$

$$v = (966.71 - 750)(1.00625)^{12t}$$



$$u = v \text{ when } t \approx 10.73 \text{ years.}$$



$$53. t = -10 \ln\left(\frac{T - 70}{98.6 - 70}\right)$$

At 9:00 A.M. we have

$$t = -10 \ln[(85.7 - 70)/(98.6 - 70)] \approx 6 \text{ hours.}$$

Thus, we can conclude that the person died 6 hours before 9 A.M., or 3:00 A.M.

$$54. t = -5.05 \ln\left(\frac{T - 40}{0 - 40}\right) \quad (t = 0 \text{ is 11 A.M.})$$

$$7 = -5.05 \ln\left(\frac{T - 40}{0 - 40}\right)$$

$$\frac{-7}{5.05} = \ln\left(\frac{T - 40}{-40}\right)$$

$$\left(\frac{T - 40}{-40}\right) = e^{-7/5.05}$$

$$T = 40 - 40e^{-7/5.05} \approx 29.998 \approx 30 < 32$$

Hence, the steaks do not thaw out in time.

55. False. The domain could be all real numbers.

56. False. See Example 5, page 380.

57. True. For the Gaussian model,  $y > 0$ .

58. True. See page 379.

$$59. 4x - 3y - 9 = 0 \Rightarrow y = \frac{4}{3}(4x - 9)$$

$$\text{Slope: } \frac{4}{3}$$

Matches (a).

$$\text{Intercepts: } (0, -3), \left(\frac{9}{4}, 0\right)$$

60. Line with intercepts (5, 0) and (0, 2).

Matches (b).

$$61. y = 25 - 2.25x$$

$$\text{Slope: } -2.25$$

Matches (d).

$$\text{Intercepts: } (0, 25), \left(\frac{100}{9}, 0\right)$$

62. Line with intercepts (2, 0) and (0, 4).

Matches (c).

$$63. f(x) = 2x^3 - 3x^2 + x - 1$$

The graph falls to the left and rises to the right.

$$64. f(x) = -4x^4 - x^2 + 5$$

Falls to the left and falls to right

$$65. g(x) = -1.6x^5 + 4x^2 - 2$$

The graph rises to the left and falls to the right.

$$66. g(x) = 7x^6 + 9.1x^5 - 3.2x^4 + 25x^3$$

Rises to left and rises to right

$$67. \begin{array}{r|rrrr} 4 & 2 & -8 & 3 & -9 \\ & & 8 & 0 & 12 \\ \hline & 2 & 0 & 3 & 3 \end{array}$$

$$\frac{2x^3 - 8x^2 + 3x - 9}{x - 4} = 2x^2 + 3 + \frac{3}{x - 4}$$

$$68. \begin{array}{r|rrrrr} -5 & 1 & 0 & 0 & -3 & 1 \\ & & -5 & 25 & -125 & 640 \\ \hline & 1 & -5 & 25 & -128 & 641 \end{array}$$

$$\frac{x^4 - 3x + 1}{x + 5} = x^3 - 5x^2 + 25x - 128 + \frac{641}{x + 5}$$

69. Answers will vary.

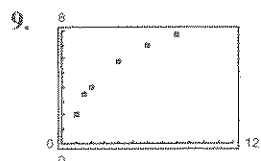
## Section 3.6 Nonlinear Models

- You should be able to use a graphing utility to find nonlinear models, including:
  - Quadratic models
  - Exponential models
  - Power models
  - Logarithmic models
  - Logistic models
- You should be able to use a scatter plot to determine which model is best.
- You should be able to determine the sum of squared differences for a model.

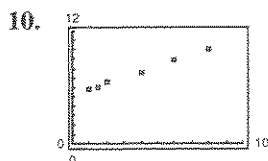
## Vocabulary Check

- $y = ax + b$
- quadratic
- $y = ax^b$
- sum, squared differences
- $y = ab^x, ae^{cx}$

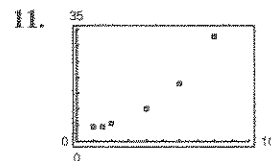
- Logarithmic model
- Linear model
- Quadratic model
- Exponential model
- Exponential model
- Logistic model
- Quadratic model
- Linear model



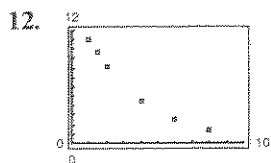
Logarithmic model



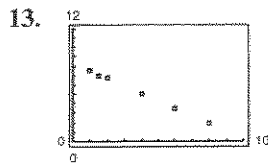
Linear model



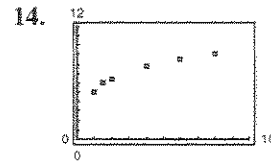
Exponential model



Exponential model



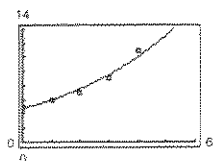
Linear model



Logarithmic model

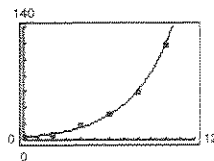
15.  $y = 4.752(1.2607)^x$

Coefficient of determination:  
0.96773



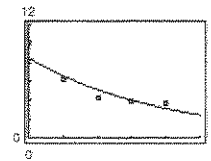
16.  $y = 3.964(1.4084)^x$

Coefficient of determination:  
0.99495

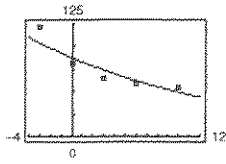


17.  $y = 8.463(0.7775)^x$

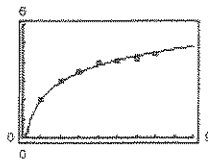
Coefficient of determination:  
0.86639



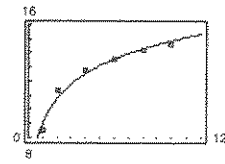
18.  $y = 87.262(0.9438)^x$

Coefficient of determination:  
0.85030

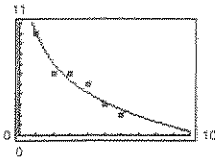
19.  $y = 2.083 + 1.257 \ln x$

Coefficient of determination:  
0.98672

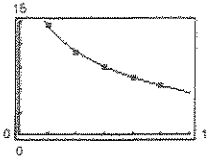
20.  $y = 9.027 + 2.537 \ln x$

Coefficient of determination:  
0.96884

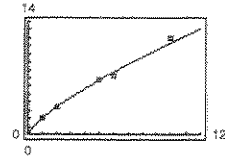
21.  $y = 9.826 - 4.097 \ln x$

Coefficient of determination:  
0.93704

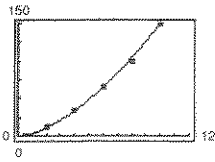
22.  $y = 20.076 - 5.027 \ln x$

Coefficient of determination:  
0.99977

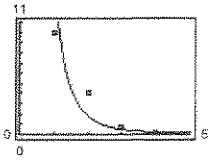
23.  $y = 1.985x^{0.760}$

Coefficient of determination:  
0.99686

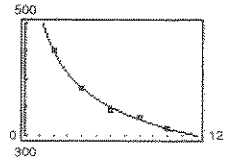
24.  $y = 3.397x^{1.650}$

Coefficient of determination:  
0.99788

25.  $y = 16.103x^{-3.174}$

Coefficient of determination:  
0.88161

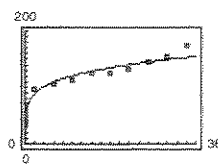
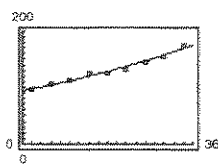
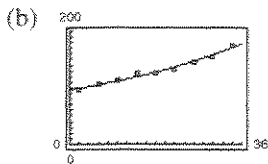
26.  $y = 525.428x^{-0.226}$

Coefficient of determination:  
0.99549

27. (a) Quadratic model:  $R = 0.031t^2 + 1.13t + 97.1$

Exponential model:  $R = 94.435(1.0174)^t$

Power model:  $R = 77.837t^{0.1918}$



(c) The exponential model fits best. Answers will vary.

(d) For 2008,  $t = 38$  and  $R \approx 181.9$  million.For 2012,  $t = 42$  and  $R \approx 194.9$  million.

Answers will vary.

28. (a) Quadratic model:
- $R = -0.0136t^2 + 0.396t + 1.01$

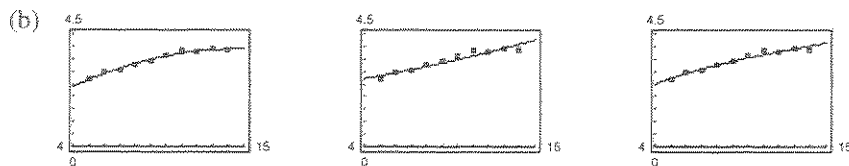
Coefficient of determination: 0.97730

Exponential model:  $R = 2.296(1.0425)^t$ 

Coefficient of determination: 0.90739

Power model:  $R = 1.480t^{0.3791}$ 

Coefficient of determination: 0.96052



- (c) The quadratic model fits best.

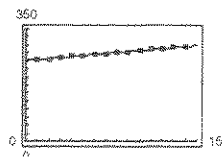
- (d) Using the quadratic model:

Year	2005	2006	2007	2008	2009	2010
Price	3.89	3.86	3.81	3.73	3.62	3.49

Answers will vary.

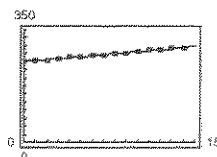
29. (a) Linear model:
- $P = 3.11t + 250.9$

Coefficient of determination: 0.99942



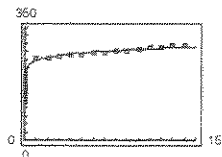
- (c) Exponential model:
- $P = 251.57(1.0114)^t$

Coefficient of determination: 0.99811



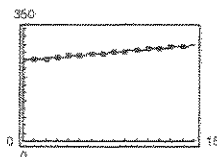
- (b) Power model:
- $P = 246.52t^{0.0587}$

Coefficient of determination: 0.90955



- (d) Quadratic model:
- $P = -0.020t^2 + 3.41t + 250.1$

Coefficient of determination: 0.99994



—CONTINUED—

- (e) The quadratic model is best because its coefficient of determination is closest to 1.

## 29. —CONTINUED—

(f) Linear model:

Year	2005	2006	2007	2008	2009	2010
Population (in millions)	297.6	300.7	303.8	306.9	310.0	313.1

Power model:

Year	2005	2006	2007	2008	2009	2010
Population (in millions)	289.0	290.1	291.1	292.1	293.0	293.9

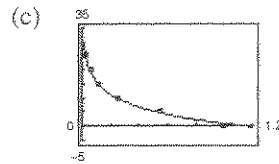
Exponential model:

Year	2005	2006	2007	2008	2009	2010
Population (in millions)	298.2	301.6	305.0	308.5	312.0	315.6

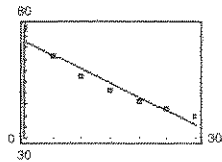
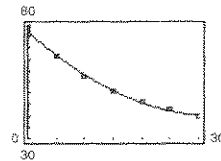
Quadratic model:

Year	2005	2006	2007	2008	2009	2010
Population (in millions)	296.8	299.5	302.3	305.0	307.7	310.3

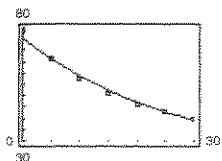
(g) and (h) Answers will vary.

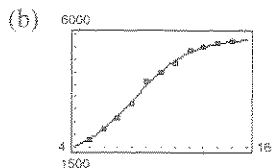
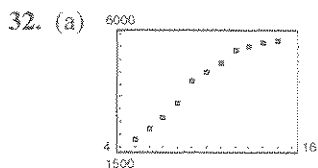
30. (a)  $h = 0$  is not in the domain of the logarithmic function.(b)  $h = 0.863 - 6.447 \ln p$ (d) For  $p = 0.75$ ,  $h \approx 2.71$  km.(e) For  $h = 13$ ,  $p \approx 0.15$  atmospheres.31. (a)  $T = -1.239t + 73.02$ 

No, the data does not appear linear.

(b)  $T = 0.034t^2 - 2.26t + 77.3$ Yes, the data appears quadratic. But, for  $t = 60$ , the graph is increasing, which is incorrect.(c) Subtracting 21 from the  $T$ -values, the exponential model is  $y = 54.438(0.9635)^t$ . Adding back 21,  $T = 54.438(0.9635)^t + 21$ .

(d) Answers will vary.

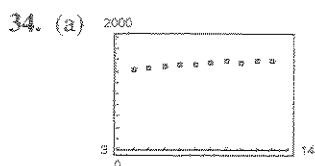




This model is a good fit.

(c) 
$$S = 1018.4 + \frac{4827.2}{1 + e^{-(t-8.1391)/1.9372}}$$

Using a graphing utility,  $t \approx 15.7$  or 2005.



(b) Linear model:  $y = 18.5x + 1365$

Quadratic model:  $-2.10x^2 + 54.2x + 1230$

Cubic model:  $y = -0.071x^3 - 0.30x^2 + 39.9x + 1265$

Power model:  $y = 1239.7 \cdot x^{0.0985}$

Exponential model:  $y = 1370.4(1.012)^x$

(c)

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Linear	1439	1458	1476	1495	1513	1532	1550	1569	1587	1606
Quadratic	1413	1449	1480	1507	1529	1548	1562	1572	1578	1580
Cubic	1415	1448	1478	1505	1529	1548	1563	1573	1578	1577
Power	1421	1453	1479	1502	1522	1539	1555	1570	1584	1596
Exponential	1437	1455	1472	1490	1508	1526	1544	1563	1581	1600

Answers will vary.

(d) For 2015,  $x = 25$  and  $y \approx 1273$  million metric tons.

35. (a) Linear model:  $y = 15.71t + 51.0$

Logarithmic model:  $y = 134.67 \ln t - 97.5$

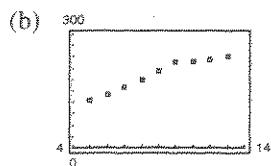
Quadratic model:  $y = -1.292t^2 + 38.96t - 45.0$

Exponential model:  $y = 85.97(1.091)^t$

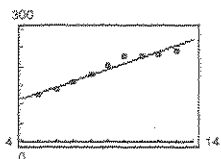
Power model:  $y = 37.27t^{0.7506}$

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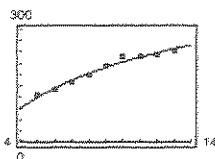
## 35. —CONTINUED—



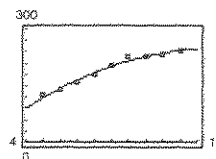
Linear model:



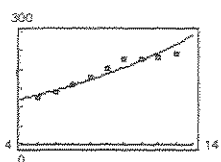
Logarithmic model:



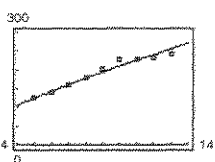
Quadratic model:



Exponential model:



Power model:



(c) Linear: 803.9

Logarithmic: 411.7

Quadratic: 289.8 (Best)

Exponential: 1611.4

Power: 667.1

(d) Linear: 0.9485

Logarithmic: 0.9736

Quadratic: 0.9814 (Best)

Exponential: 0.9274

Power: 0.9720

(e) Quadratic model is best.

36. Answers will vary.

37. True

 38. False. Write  $b$  as  $b = e^{\ln b}$ .

Then,

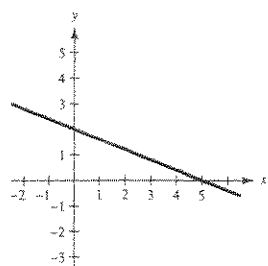
$$y = ab^x = ae^{(\ln b)x} = ae^{cx}.$$

39.  $2x + 5y = 10$

$$5y = -2x + 10$$

$$y = -\frac{2}{5}x + 2$$

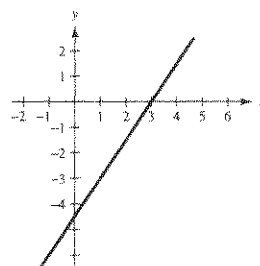
 Slope:  $-\frac{2}{5}$ 

 y-intercept:  $(0, 2)$ 


40.  $3x - 2y = 9$

$$y = \frac{3}{2}x - \frac{9}{2}$$

 Slope:  $\frac{3}{2}$ 

 y-intercept:  $(0, -\frac{9}{2})$ 


41.  $1.2x + 3.5y = 10.5$

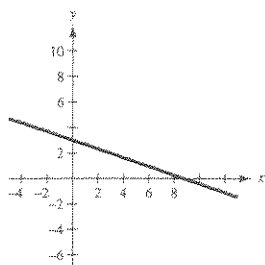
$$35y = -12x + 105$$

$$y = -\frac{12}{35}x + \frac{105}{35}$$

$$= -\frac{12}{35}x + 3$$

Slope:  $-\frac{12}{35}$

y-intercept:  $(0, 3)$



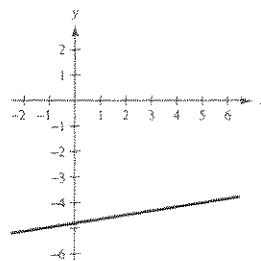
42.  $0.4x - 2.5y = 12.0$

$$25y = 4x - 120$$

$$y = \frac{4}{25}x - \frac{24}{5}$$

Slope:  $\frac{4}{25} = 0.16$

y-intercept:  $(0, -\frac{24}{5}) = (0, -4.8)$



### Review Exercises for Chapter 3

1.  $(1.45)^{2\pi} \approx 10.3254$

2.  $7^{-\sqrt{11}} \approx 0.002$

$$3. 60^{2(-1.1)} = 60^{-2.2} \\ \approx 0.0001225 \approx 0.0$$

4.  $25^{-3(3/2)} \approx 5.12 \times 10^{-7} \approx 0$

5.  $e^8 \approx 2980.958$

6.  $5e^{\sqrt{3}} \approx 46.7823$

7.  $e^{-(-2.1)} \approx e^{2.1} \approx 8.1662$

8.  $-4e^{(-3/5)} \approx -2.1952$

9.  $f(x) = 4^x$

Intercept:  $(0, 1)$ Horizontal asymptote:  $x$ -axisIncreasing on:  $(-\infty, \infty)$ 

Matches graph (c).

10.  $f(x) = 4^{-x}$

Intercept:  $(0, 1)$ Horizontal asymptote:  $x$ -axisDecreasing on:  $(-\infty, \infty)$ 

Matches graph (d).

11.  $f(x) = -4^x$

Intercept:  $(0, -1)$ Horizontal asymptote:  $x$ -axisDecreasing on:  $(-\infty, \infty)$ 

Matches graph (b).

12.  $f(x) = 4^x + 1$

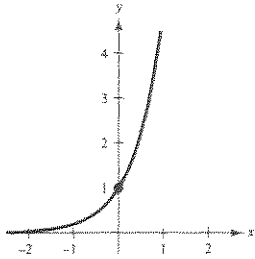
Intercept:  $(0, 2)$ Horizontal asymptote:  $y = 1$ Increasing on:  $(-\infty, \infty)$ 

Matches graph (a).



13.  $f(x) = 6^x$

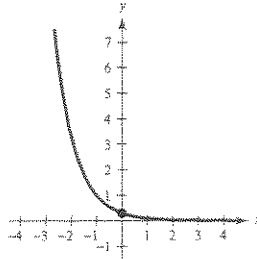
Intercept: (0, 1)

Horizontal asymptote:  $x$ -axisIncreasing on:  $(-\infty, \infty)$ 

14.  $f(x) = 0.3^{x+1} = \left(\frac{3}{10}\right)^{x+1}$

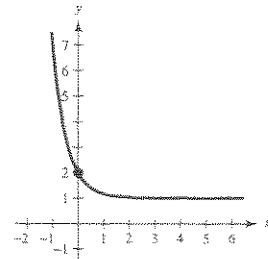
Horizontal asymptote:  $y = 0$ 

Intercept: (0, 0.3)

Decreasing on  $(-\infty, \infty)$ 

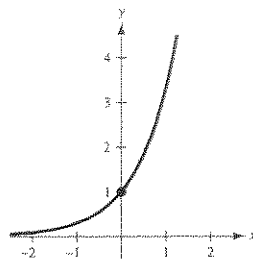
15.  $g(x) = 1 + 6^{-x}$

Intercept: (0, 2)

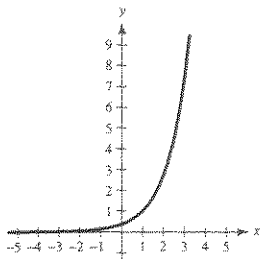
Horizontal asymptote:  $y = 1$ Decreasing on:  $(-\infty, \infty)$ 

16.  $g(x) = 0.3^{-x}$

$x$	-2	-1	0	1	2
$y$	0.09	0.3	1	$3\frac{1}{3}$	$11\frac{1}{9}$



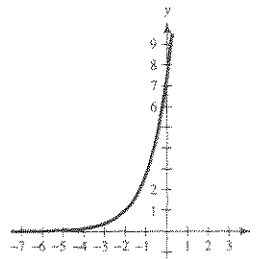
17.



$h(x) = e^{x-1}$

Horizontal asymptote:  $y = 0$ 

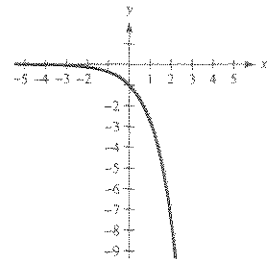
18.



$f(x) = e^{x+2}$

Horizontal asymptote:  $y = 0$ 

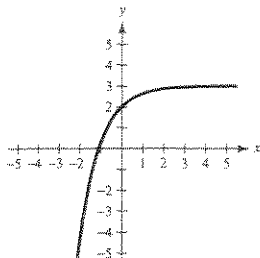
19.



$h(x) = -e^x$

Horizontal asymptote:  $y = 0$ 

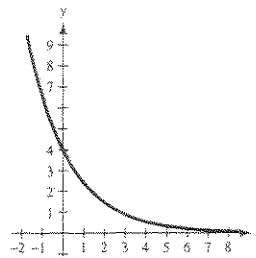
20.



$f(x) = 3 - e^{-x}$

Horizontal asymptote:  $y = 3$ 

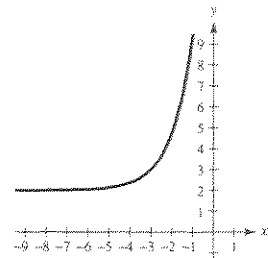
21.



$f(x) = 4e^{-0.5x}$

Horizontal asymptote:  $y = 0$ 

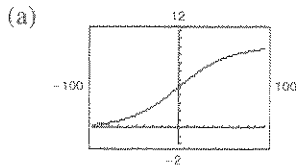
22.



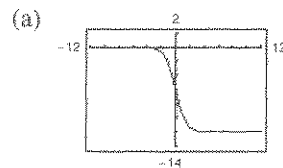
$f(x) = 2 + e^{x+3}$

Horizontal asymptote:  $y = 2$

23.  $f(x) = \frac{10}{1 + 2e^{-0.05x}}$

(b) Horizontal asymptotes:  $y = 0$ ,  $y = 10$ 

24.  $f(x) = \frac{-12}{1 + 4^{-x}}$

(b) Horizontal asymptotes:  $y = 0$ ,  $y = -12$ 

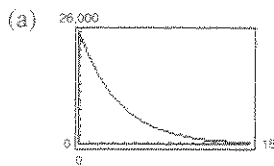
25.  $A = Pe^{rt} = 10,000e^{0.08t}$

$t$	1	10	20	30	40	50
$A$	10,832.87	22,255.41	49,530.32	110,231.76	245,325.30	545,981.50

26.  $r = 3\% = 0.03$ ,  $A = 10,000e^{0.03t}$

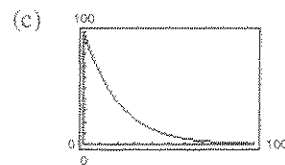
$t$	1	10	20	30	40	50
$A$	10,305	13,499	18,221	24,596	33,201	44,817

27.  $V(t) = 26,000\left(\frac{3}{4}\right)^t$

(b) For  $t = 2$ ,  $V(2) = \$14,625$ .

(c) The car depreciates most rapidly at the beginning, which is realistic.

28.  $Q = 100\left(\frac{1}{2}\right)^{t/14}$

(a) When  $t = 0$ ,  $Q = 100$  grams.(b) When  $t = 10$ ,  $Q = 100\left(\frac{1}{2}\right)^{10/14} \approx 60.95$  grams.

29.  $\log_5 125 = 3$

$5^3 = 125$

30.  $\log_6 36 = 2$

$6^2 = 36$

31.  $\log_{64} 2 = \frac{1}{6}$

$64^{1/6} = 2$

32.  $\log_{10}\left(\frac{1}{100}\right) = -2$

$10^{-2} = \frac{1}{100}$

33.  $\ln e^4 = 4$

$e^4 = e^4$

34.  $\ln \sqrt{e^3} = \frac{3}{2}$

$e^{3/2} = \sqrt{e^3}$

35.  $4^3 = 64$

$\log_4 64 = 3$

36.  $3^5 = 243$

$\log_3 243 = 5$

37.  $25^{3/2} = 125$

$\log_{25} 125 = \frac{3}{2}$

38.  $12^{-1} = \frac{1}{12}$

$\log_{12}\left(\frac{1}{12}\right) = -1$

39.  $\left(\frac{1}{2}\right)^{-3} = 8$

$\log_{1/2} 8 = -3$

40.  $\left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$

$\log_{2/3}\left(\frac{9}{4}\right) = -2$

41.  $e^7 = 1096.6331\dots$

$\ln 1096.6331\dots = 7$

42.  $e^{-3} = 0.0497\dots$

$\ln 0.0497\dots = -3$

43.  $\log_6 216 = \log_6 6^3$

$= 3 \log_6 6$

$= 3$

44.  $\log_7 1 = 0$

45.  $\log_4\left(\frac{1}{4}\right) = \log_4(4^{-1})$   
 $= -\log_4 4$   
 $= -1$

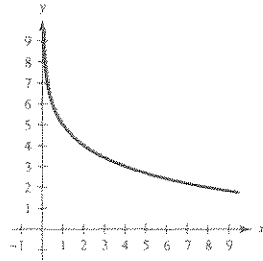
46.  $\log_{10} 0.001 = \log_{10} 10^{-3}$   
 $= -3$

47.  $g(x) = -\log_2 x + 5 = 5 - \frac{\ln x}{\ln 2}$

Domain:  $x > 0$

Vertical asymptote:  $x = 0$

x-intercept: (32, 0)



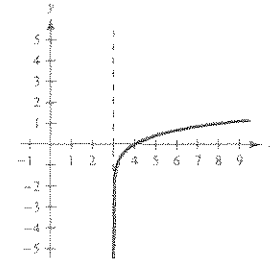
48.  $g(x) = \log_5(x - 3)$

Vertical asymptote:  $x = 3$

Intercept: (4, 0)

Domain:  $x > 3$

$x$	3.2	4	8	28
$y$	-1	0	1	2

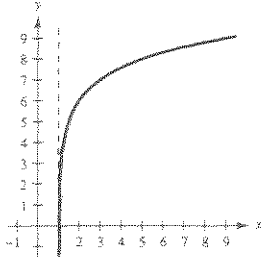


49.  $f(x) = \log_2(x - 1) + 6 = 6 + \frac{\ln(x - 1)}{\ln(2)}$

Domain:  $x > 1$

Vertical asymptote:  $x = 1$

x-intercept: (1.016, 0)



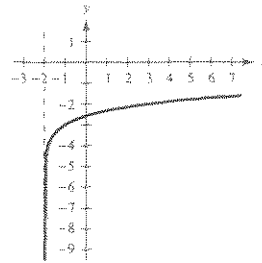
50.  $f(x) = \log_5(x + 2) - 3$

Vertical asymptote:  $x = -2$

Intercept: (123, 0)

Domain:  $x > -2$

$x$	-1.8	-1	3	23
$y$	-4	-3	-2	-1



51.  $\ln(21.5) \approx 3.068$

52.  $\ln(0.98) \approx -0.020$

53.  $\ln\sqrt{6} \approx 0.896$

54.  $\ln\left(\frac{2}{5}\right) \approx -0.916$

55.  $\log_5 3 = \log_5 x$

$3 = x$

56.  $\log_2 8 = x$

$2^x = 8$

$x = 3$

57.  $\log_9 x = \log_9 3^{-2}$

$x = 3^{-2} = \frac{1}{9}$

58.  $\log_4 4^3 = x$

$4^x = 4^3$

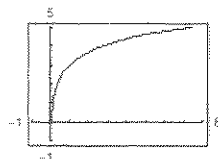
$x = 3$

59.  $f(x) = \ln x + 3$

Domain:  $(0, \infty)$

Vertical asymptote:  $x = 0$

x-intercept: (0.05, 0)

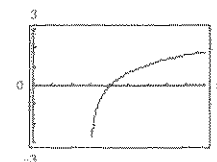


60.  $f(x) = \ln(x - 3)$

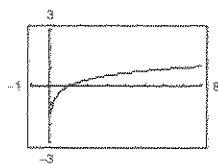
Domain:  $(3, \infty)$

Vertical asymptote:  $x = 3$

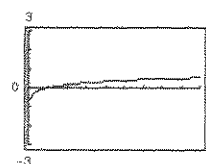
Intercept: (4, 0)



61.  $h(x) = \frac{1}{2} \ln x$

Domain:  $x > 0$ Vertical asymptote:  $x = 0$  $x$ -intercept:  $(1, 0)$ 

62.  $f(x) = \frac{1}{4} \ln x$

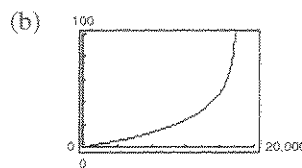
Domain:  $(0, \infty)$ Vertical asymptote:  $x = 0$ Intercept:  $(1, 0)$ 

63.  $t = 50 \log_{10} \frac{18,000}{18,000 - h}$

(a)  $0 \leq h < 18,000$

(c) The plane climbs at a faster rate as it approaches its absolute ceiling.

(d) If  $h = 4000$ ,  $t = 50 \log_{10} \frac{18,000}{18,000 - 4000} \approx 5.46$  minutes.

Vertical asymptote:  $h = 18,000$ 

64.  $t = 12.542 \ln \left( \frac{x}{x - 1000} \right), x > 1000$

(a) For  $x = 1254.68$ ,  $t \approx 20$  years.(b) For  $x = 1254.68$ ,  $t = 20$ , the total amount paid is  $(1254.68)(20)(12) = \$301,123.20$ .The interest is  $301,123.20 - 150,000.00 = \$151,123.20$ .

65.  $\log_4 9 = \frac{\log_{10} 9}{\log_{10} 4} \approx 1.585$

$\log_4 9 = \frac{\ln 9}{\ln 4} \approx 1.585$

66.  $\log_{1/2} 5 = \frac{\log_{10} 5}{\log_{10}(1/2)} \approx -2.322$

$\log_{1/2} 5 = \frac{\ln 5}{\ln(1/2)} \approx -2.322$

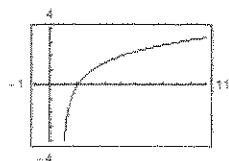
67.  $\log_{12} 200 = \frac{\log_{10} 200}{\log_{10} 12} \approx 2.132$

$\log_{12} 200 = \frac{\ln 200}{\ln 12} \approx 2.132$

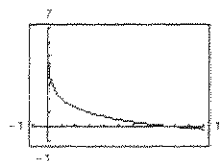
68.  $\log_3 0.28 = \frac{\log_{10} 0.28}{\log_{10} 3} \approx -1.159$

$\log_3 0.28 = \frac{\ln 0.28}{\ln 3} \approx -1.159$

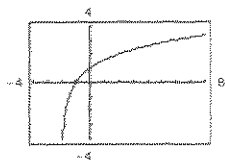
69.  $f(x) = \log_2(x - 1) = \frac{\ln(x - 1)}{\ln 2}$



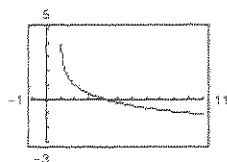
70.  $f(x) = 2 - \log_3 x = 2 - \frac{\ln x}{\ln 3}$



$$71. f(x) = -\log_{1/2}(x+2) = -\frac{\ln(x+2)}{\ln(1/2)} = \frac{\ln(x+2)}{\ln 2}$$



$$72. f(x) = \log_{1/3}(x-1) + 1 = \frac{\ln(x-1)}{\ln(1/3)} + 1 = -\frac{\ln(x-1)}{\ln 3} + 1$$



$$\begin{aligned} 73. \log_b 9 &= \log_b 3^2 \\ &= 2 \log_b 3 \\ &= 2(0.5646) \\ &= 1.1292 \end{aligned}$$

$$\begin{aligned} 74. \log_b \left(\frac{4}{9}\right) &= \log_b 2^2 - \log_b 3^2 \\ &= 2 \log_b 2 - 2 \log_b 3 \\ &= 2(0.3562) - 2(0.5646) \\ &= -0.4168 \end{aligned}$$

$$\begin{aligned} 75. \log_b \sqrt{5} &= \log_b 5^{1/2} \\ &= \frac{1}{2} \log_b 5 \\ &= \frac{1}{2}(0.8271) \\ &= 0.41355 \end{aligned}$$

$$\begin{aligned} 76. \log_b 50 &= \log_b [2 \cdot 5^2] \\ &= \log_b 2 + 2 \log_b 5 \\ &= 0.3562 + 2(0.8271) \\ &= 2.0104 \end{aligned}$$

$$\begin{aligned} 77. \ln(5e^{-2}) &= \ln 5 + \ln e^{-2} \\ &= \ln 5 - 2 \ln e \\ &= \ln 5 - 2 \end{aligned}$$

$$\begin{aligned} 78. \ln \sqrt{e^5} &= \ln e^{5/2} \\ &= \frac{5}{2} \ln e = \frac{5}{2} \end{aligned}$$

$$\begin{aligned} 79. \log_{10} 200 &= \log_{10}(2 \cdot 100) \\ &= \log_{10} 2 + \log_{10} 10^2 \\ &= \log_{10} 2 + 2 \end{aligned}$$

$$\begin{aligned} 80. \log_{10} 0.002 &= \log_{10}(2 \cdot 10^{-3}) \\ &= \log_{10} 2 + \log_{10} 10^{-3} \\ &= \log_{10} 2 - 3 \end{aligned}$$

$$81. \log_5 5x^2 = \log_5 5 + \log_5 x^2 = 1 + 2 \log_5 x$$

$$82. \log_4(3xy^2) = \log_4 3 + \log_4 x + 2 \log_4 y$$

$$\begin{aligned} 83. \log_{10} \frac{5\sqrt{y}}{x^2} &= \log_{10} 5\sqrt{y} - \log_{10} x^2 \\ &= \log_{10} 5 + \log_{10} \sqrt{y} - \log_{10} x^2 \\ &= \log_{10} 5 + \frac{1}{2} \log_{10} y - 2 \log_{10} x \end{aligned}$$

$$\begin{aligned} 84. \ln \frac{\sqrt{x}}{4} &= \ln x^{1/2} - \ln 4 = \frac{1}{2} \ln x - \ln 4 \\ &= \frac{1}{2} \ln x - 2 \ln 2 \end{aligned}$$

$$\begin{aligned} 85. \ln \left( \frac{x+3}{xy} \right) &= \ln(x+3) - \ln(xy) \\ &= \ln(x+3) - \ln x - \ln y \end{aligned}$$

$$\begin{aligned} 86. \ln \frac{xy^5}{\sqrt{z}} &= \ln x + \ln y^5 - \ln z^{1/2} \\ &= \ln x + 5 \ln y - \frac{1}{2} \ln z \end{aligned}$$

$$87. \log_2 5 + \log_2 x = \log_2 5x$$

$$\begin{aligned} 88. \log_6 y - 2 \log_6 z &= \log_6 y - \log_6 z^2 \\ &= \log_6 \frac{y}{z^2} \end{aligned}$$

$$\begin{aligned} 89. \frac{1}{2} \ln(2x-1) - 2 \ln(x+1) &= \ln \sqrt{2x-1} - \ln(x+1)^2 \\ &= \ln \frac{\sqrt{2x-1}}{(x+1)^2} \end{aligned}$$

$$90. 5 \ln(x-2) - \ln(x+2) - 3 \ln(x) = \ln(x-2)^5 - \ln(x+2) - \ln(x)^3$$

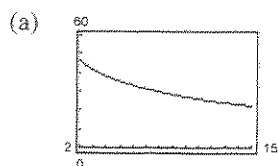
$$= \ln\left(\frac{(x-2)^5}{(x+2)x^3}\right)$$

$$91. \ln 3 + \frac{1}{3} \ln(4-x^2) - \ln x = \ln\left[\frac{3(4-x^2)^{1/3}}{x}\right] = \ln\left[\frac{3\sqrt[3]{4-x^2}}{x}\right]$$

$$92. 3[\ln x - 2 \ln(x^2+1)] + 2 \ln 5 = \ln x^3 - \ln(x^2+1)^6 + \ln 5^2$$

$$= \ln \frac{25x^3}{(x^2+1)^6}$$

$$93. s = 25 - \frac{13 \ln(h/12)}{\ln 3}$$



(b)

$h$	4	6	8	10	12	14
$s$	38	33.2	29.8	27.2	25	23.2

(c) As the depth increases, the number of miles of roads cleared decreases.

$$94. f(t) = 85 - 14 \log_{10}(t+1)$$

$$71 = 85 - 14 \log_{10}(t+1)$$

$$\log_{10}(t+1) = 1$$

$$t = 9 \text{ months}$$

$$95. 8^x = 512 = 8^3 \Rightarrow x = 3$$

$$96. 3^x = 729 = 3^6 \Rightarrow x = 6$$

$$97. 6^x = \frac{1}{216} = \frac{1}{6^3} = 6^{-3} \Rightarrow x = -3$$

$$98. 6^{x-2} = 1296 = 6^4 \Rightarrow x-2 = 4 \Rightarrow x = 6$$

$$99. 2^{x+1} = \frac{1}{16}$$

$$2^{x+1} = 2^{-4}$$

$$x+1 = -4$$

$$x = -5$$

$$100. 4^{x/2} = 64$$

$$4^{x/2} = 4^3$$

$$\frac{x}{2} = 3$$

$$x = 6$$

$$101. \log_7 x = 4 \Rightarrow x = 7^4 = 2401$$

$$102. \log_x 243 = 5 \Rightarrow x^5 = 243 = 3^5 \Rightarrow x = 3$$

$$103. \log_2(x-1) = 3$$

$$2^3 = x-1$$

$$x = 9$$

$$104. \log_5(2x+1) = 2$$

$$5^2 = 2x+1$$

$$2x = 24$$

$$x = 12$$

$$105. \ln x = 4$$

$$x = e^4 \approx 54.598$$

106.  $\ln x = -3$

$$x = e^{-3} \approx 0.0498$$

107.  $\ln(x - 1) = 2$

$$e^2 = x - 1$$

$$x = 1 + e^2$$

108.  $\ln(2x + 1) = -4$

$$e^{-4} = 2x + 1$$

$$x = \frac{e^{-4} - 1}{2}$$

$$= \frac{1 - e^4}{2e^4}$$

109.  $3e^{-5x} = 132$

$$e^{-5x} = 44$$

$$-5x = \ln 44$$

$$x = -\frac{\ln 44}{5} \approx -0.757$$

110.  $14e^{3x+2} = 560$

$$e^{3x+2} = 40$$

$$\ln e^{3x+2} = \ln 40$$

$$3x + 2 = \ln 40$$

$$x = \frac{(\ln 40) - 2}{3} \approx 0.563$$

111.  $2^x + 13 = 35$

$$2^x = 22$$

$$x \ln 2 = \ln 22$$

$$x = \frac{\ln 22}{\ln 2} \approx 4.459$$

112.  $6^x - 28 = -8$

$$6^x = 20$$

$$x \ln 6 = \ln 20$$

$$x = \frac{\ln 20}{\ln 6} \approx 1.672$$

113.  $-4(5^x) = -68$

$$5^x = 17$$

$$x \ln 5 = \ln 17$$

$$x = \frac{\ln 17}{\ln 5} \approx 1.760$$

114.  $2(12^x) = 190$

$$12^x = 95$$

$$x \ln 12 = \ln 95$$

$$x = \frac{\ln 95}{\ln 12} \approx 1.833$$

115.  $2e^{x-3} - 1 = 4$

$$2e^{x-3} = 5$$

$$e^{x-3} = \frac{5}{2}$$

$$x - 3 = \ln\left(\frac{5}{2}\right)$$

$$x = 3 + \ln\left(\frac{5}{2}\right) \approx 3.916$$

116.  $-e^{x/2} + 1 = \frac{1}{2}$

$$e^{x/2} = \frac{1}{2}$$

$$\frac{x}{2} = \ln\left(\frac{1}{2}\right) = -\ln 2$$

$$x = -2 \ln 2$$

$$= -\ln 4 \approx -1.386$$

117.  $e^{2x} - 7e^x + 10 = 0$

$$(e^x - 5)(e^x - 2) = 0$$

$$e^x = 5 \Rightarrow x = \ln 5 \approx 1.609$$

$$e^x = 2 \Rightarrow x = \ln 2 \approx 0.693$$

118.  $e^{2x} - 6e^x + 8 = 0$

$$(e^x - 4)(e^x - 2) = 0$$

$$e^x = 4 \quad \text{or} \quad e^x = 2$$

$$x = \ln 4 \quad \text{or} \quad x = \ln 2$$

$$x \approx 1.386 \quad x \approx 0.693$$

119.  $\ln 3x = 8.2$

$$3x = e^{8.2}$$

$$x = \frac{e^{8.2}}{3} \approx 1213.650$$

120.  $\ln 5x = 7.2$

$$5x = e^{7.2}$$

$$x = \frac{1}{5}e^{7.2} \approx 267.886$$

121.  $\ln x - \ln 3 = 2$

$$\ln \frac{x}{3} = 2$$

$$\frac{x}{3} = e^2$$

$$x = 3e^2 \approx 22.167$$

122.  $\ln x - \ln 5 = 4$

$$\ln\left(\frac{x}{5}\right) = 4$$

$$e^4 = \frac{x}{5}$$

$$x = 5e^4 \approx 272.991$$

123.  $\ln \sqrt{x+1} = 2$

$$\frac{1}{2} \ln(x+1) = 2$$

$$\ln(x+1) = 4$$

$$x+1 = e^4$$

$$x = e^4 - 1$$

$$\approx 53.598$$

124.  $\ln \sqrt{x+8} = 3$

$$\frac{1}{2} \ln(x+8) = 3$$

$$\ln(x+8) = 6$$

$$x+8 = e^6$$

$$x = e^6 - 8 \approx 395.429$$

125.  $\log_4(x-1) = \log_4(x-2) - \log_4(x+2)$

$$\log_4(x-1) = \log_4\left(\frac{x-2}{x+2}\right)$$

$$x-1 = \frac{x-2}{x+2}$$

$$(x-1)(x+2) = x-2$$

$$x^2 + x - 2 = x - 2$$

$$x^2 = 0$$

$$x = 0 \text{ (extraneous)}$$

No solution

126.  $\log_5(x+2) - \log_5(x) = \log_5(x+5)$

$$\log_5\left(\frac{x+2}{x}\right) = \log_5(x+5)$$

$$\frac{x+2}{x} = x+5$$

$$x+2 = x^2 + 5x$$

$$x^2 + 4x - 2 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-2)}}{2} = -2 \pm \sqrt{6}$$

$$x = -2 + \sqrt{6} \approx 0.449$$

(Other zero is extraneous.)

127.  $\log_{10}(1-x) = -1$

$$10^{-1} = 1-x$$

$$x = 1 - 10^{-1} = 0.9$$

128.  $\log_{10}(-x-4) = 2$

$$-x-4 = 10^2 = 100$$

$$-x = 104$$

$$x = -104$$

129.  $xe^x + e^x = 0$

$$(x+1)e^x = 0$$

$$x+1 = 0 \text{ (since } e^x \neq 0)$$

$$x = -1$$

130.  $2xe^{2x} + e^{2x} = 0$

$$(2x+1)e^{2x} = 0$$

$$2x+1 = 0 \text{ (since } e^{2x} \neq 0)$$

$$x = -\frac{1}{2}$$

131.  $x \ln x + x = 0$

$$x(\ln x + 1) = 0$$

$$\ln x + 1 = 0 \text{ (since } x > 0)$$

$$\ln x = -1$$

$$x = e^{-1} = \frac{1}{e} \approx 0.368$$

132.  $\frac{1 - \ln x}{x^2} = 0$

$$1 - \ln x = 0 \text{ (since } x > 0)$$

$$\ln x = 1$$

$$x = e \approx 2.718$$

133.  $3(7550) = 7550e^{0.0725t}$

$$3 = e^{0.0725t}$$

$$\ln 3 = 0.0725t$$

$$t = \frac{\ln 3}{0.0725} \approx 15.2 \text{ years}$$



134.  $p = 500 - 0.5e^{0.004x}$

(a)  $p = 450$

$$450 = 500 - 0.5e^{0.004x}$$

$$0.5e^{0.004x} = 50$$

$$e^{0.004x} = 100$$

$$0.004x = \ln 100$$

$$x \approx 1151 \text{ units}$$

(b)  $p = 400$

$$400 = 500 - 0.5e^{0.004x}$$

$$0.5e^{0.004x} = 100$$

$$e^{0.004x} = 200$$

$$0.004x = \ln 200$$

$$x \approx 1325 \text{ units}$$

135.  $y = 3e^{-2x/3}$

Decreasing exponential

Matches graph (e).

136.  $y = 4e^{2x/3}$

Intercept: (0, 4)

Increasing

Matches (b).

137.  $y = \ln(x + 3)$

Logarithmic function shifted to left

Matches graph (f).

138.  $y = 7 - \log(x + 3)$

Vertical asymptote:  $x = -3$

Decreasing

Matches (d).

139.  $y = 2e^{-(x+4)^2/3}$

Gaussian model

Matches graph (a).

140.  $y = \frac{6}{1 + 2e^{-2x}}$

Logistic model

Matches (c).

141.  $y = ae^{bx}$

$$2 = ae^{b(0)} \Rightarrow a = 2$$

$$3 = 2e^{b(4)}$$

$$1.5 = e^{4b}$$

$$\ln 1.5 = 4b \Rightarrow b \approx 0.1014$$

$$\text{Thus, } y = 2e^{0.1014x}.$$

142.  $y = ae^{bx}$

$$2 = ae^{b(0)} \Rightarrow a = 2$$

$$1 = 2e^{b(5)} \Rightarrow \frac{1}{2} = e^{5b} \Rightarrow 5b = \ln \frac{1}{2} \Rightarrow b = \frac{1}{5} \ln \frac{1}{2} = -\frac{1}{5} \ln 2 \approx -0.1386$$

$$y = 2e^{-0.1386x}$$

143.  $y = ae^{bx}$

$$\frac{1}{2} = ae^{b(0)} \Rightarrow a = \frac{1}{2}$$

$$5 = \frac{1}{2}e^{b(5)}$$

$$10 = e^{5b}$$

$$\ln 10 = 5b \Rightarrow b \approx 0.4605$$

$$\text{Thus, } y = \frac{1}{2}e^{0.4605x}.$$

144.  $y = ae^{bx}$

$$4 = ae^{b(0)} = a \Rightarrow a = 4$$

$$\frac{1}{2} = 4e^{b(5)}$$

$$\frac{1}{8} = e^{5b}$$

$$\ln \frac{1}{8} = 5b \Rightarrow b = -\frac{\ln 8}{5} \approx -0.4159$$

$$\Rightarrow y = 4e^{-0.4159x}$$

145.  $P = 361e^{kt}$

 $t = 0$  corresponds to 2000. $(-20, 215)$ :

$$215 = 361e^{k(-20)}$$

$$\frac{215}{361} = e^{-20k}$$

$$-20k = \ln\left(\frac{215}{361}\right)$$

$$k = -\frac{1}{20} \ln\left(\frac{215}{361}\right) = \frac{1}{20} \ln\left(\frac{361}{215}\right) \approx 0.02591$$

$$P = 361e^{0.02591t}$$

For 2020,  $P(20) = 361e^{0.02591(20)} \approx 606.1$  or

606,100 population in 2020.

146.  $\frac{1}{2}P = Pe^{k(245,500)}$

$$\ln\left(\frac{1}{2}\right) = 245,500k \Rightarrow k = \frac{-\ln 2}{245,500} \approx -2.8234 \times 10^{-6}$$

After 5000 years,

$$A = e^{k(5000)} \approx 0.98598 \text{ or } 98.6\% \text{ remains.}$$

147. (a)  $20,000 = 10,000e^{r(12)}$

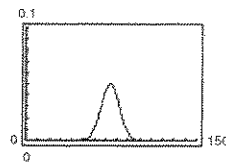
$$2 = e^{12r}$$

$$\ln 2 = 12r$$

$$r = \frac{\ln 2}{12} \approx 0.0578 \text{ or } 5.78\%$$

(b)  $10,000e^{0.0578(1)} \approx \$10,595.03$

148. (a)



$$y = 0.0499e^{-(x-74)^2/128}$$

(b) The average score corresponds to the maximum, 74.

149. (a)  $50 = \frac{158}{1 + 5.4e^{-0.12t}}$

$$1 + 5.4e^{-0.12t} = \frac{158}{50}$$

$$5.4e^{-0.12t} = \frac{108}{50}$$

$$e^{-0.12t} = \frac{108}{50(5.4)}$$

$$-0.12t = \ln \frac{108}{270}$$

$$t = \frac{\ln(108/270)}{-0.12} \approx 7.6 \text{ weeks}$$

(b) Similarly:

$$75 = \frac{158}{1 + 5.4e^{-0.12t}}$$

$$1 + 5.4e^{-0.12t} = \frac{158}{75}$$

$$e^{-0.12t} = 0.20494$$

$$-0.12t = \ln(0.20494)$$

$$t \approx 13.2 \text{ weeks}$$

150.  $R = \log_{10}\left(\frac{I}{I_0}\right) = \log_{10}(I) \Rightarrow I = 10^R$

(a)  $I = 10^{8.4} \approx 251,188,643$

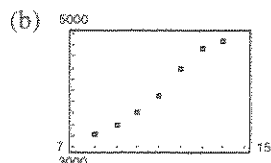
(b)  $I = 10^{6.85} \approx 7,079,458$

(c)  $I = 10^{9.1} \approx 1,258,925,412$

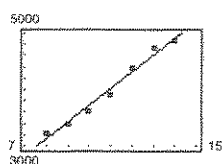
151. Logistic model

152. Linear model

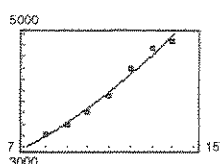
153. Logarithmic model

154. Quadratic model  
(or exponential)155. (a) Linear model:  $y = 297.8t + 739$ ; 0.97653Quadratic model:  $y = 11.79t^2 + 38.5t + 2118$ ; 0.98112Exponential model:  $y = 1751.5(1.077)^t$ ; 0.98225Logarithmic model:  $y = 3169.8 \ln t - 3532$ ; 0.95779Power model:  $y = 598.1t^{0.7950}$ ; 0.97118

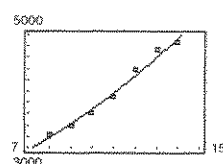
Linear model:



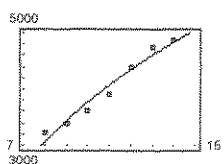
Quadratic model:



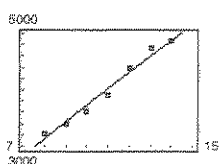
Exponential model:



Logarithmic model:



Power model:



(c) The exponential model is best because its coefficient of determination is closest to 1.

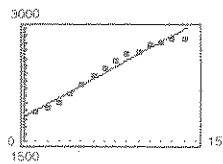
Answers will vary.

(d) For 2010,  $t = 20$  and  $y \approx \$7722$  million.(e)  $y = 5250$  when  $t \approx 14.8$ , or 2004.156. (a) Linear model:  $y = 82.9t + 1825$ ; 0.96953Quadratic model:  $y = -3.35t^2 + 133.2t + 1691$ ; 0.98982Exponential model:  $y = 1865(1.0354)^t$ ; 0.95481Logarithmic model:  $y = 1663 + 435.5 \ln t$ ; 0.91664Power model:  $y = 1733t^{0.1862}$ ; 0.93412

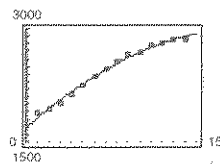
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## 156. —CONTINUED—

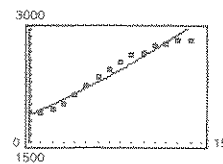
(b) Linear model:



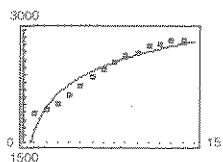
Quadratic model:



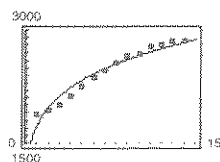
Exponential model:



Logarithmic model:



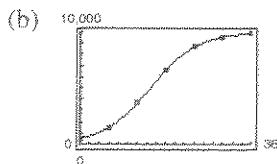
Power model:



(c) The quadratic model is best because its coefficient of determination is closest to 1.

(d) For 2010,  $t = 20$  and  $y \approx 3015$  thousand.(e)  $y = 3000$  when  $t \approx 17.8$ , or 2007.

157. (a)  $P = \frac{9999.887}{1 + 19.0e^{-0.2x}}$



(c) The model is a good fit.

(d) The limiting size is  $\frac{9999.887}{1 + 0} \approx 10,000$  fish.

158.  $P = 56.8e^{0.001603t}$

(a)

Year	1990	1991	1992	1993	1994	1995
P	56.8	56.9	57	57.1	57.2	57.3

Year	1996	1997	1998	1999	2000	2001
P	57.3	57.4	57.5	57.6	57.7	57.8

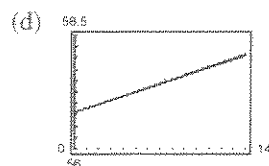
Year	2002	2003	2004	2005
P	57.9	58.0	58.1	58.2

(b) Slope =  $\frac{58.2 - 56.8}{15 - 0} \approx 0.09$

$$y - 56.8 = 0.09(t - 0)$$

$$y = 0.09t + 56.8, \text{ Linear model}$$

(c) Slope is 0.09. The population increases by 90,000 people each year.



Answers will vary.

159. True; by the Inverse Properties,  $\log_b b^{2x} = 2x$ .

$$160. e^{x-1} = e^x \cdot e^{-1} = \frac{e^x}{e}$$

True (by Properties of exponents).

161. False;  $\ln x + \ln y = \ln(xy) \neq \ln(x + y)$

$$162. \ln(x + y) \neq \ln(x \cdot y)$$

False

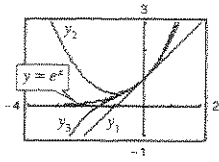
$$\ln(x \cdot y) = \ln x + \ln y \neq \ln(x + y)$$

163. False. The domain of  $f(x) = \ln(x)$  is  $x > 0$ .

$$164. \text{ True. } \ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

165. Since  $1 < \sqrt{2} < 2$ ,  $2^1 < 2^{\sqrt{2}} < 2^2 \Rightarrow 2 < 2^{\sqrt{2}} < 4$ .

166. (a)



(b) Pattern  $\sum_{i=0}^n \frac{x^i}{i!}$

$$y_4 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

The graph of  $y_4$  closely approximates  $y = e^x$  near  $(0, 1)$ .

**Chapter 3 Practice Test**

1. Solve for  $x$ :  $x^{3/5} = 8$
2. Solve for  $x$ :  $3^{x-1} = \frac{1}{81}$
3. Graph  $f(x) = 2^{-x}$  by hand.
4. Graph  $g(x) = e^x + 1$  by hand.
5. If \$5000 is invested at 9% interest, find the amount after three years if the interest is compounded  
(a) monthly. (b) quarterly. (c) continuously.
6. Write the equation in logarithmic form:  $7^{-2} = \frac{1}{49}$
7. Solve for  $x$ :  $x - 4 = \log_2 \frac{1}{64}$
8. Given  $\log_b 2 = 0.3562$  and  $\log_b 5 = 0.8271$ , evaluate  $\log_b \sqrt[4]{8/25}$ .
9. Write  $5 \ln x - \frac{1}{2} \ln y + 6 \ln z$  as a single logarithm.
10. Using your calculator and the change of base formula, evaluate  $\log_9 28$ .
11. Use your calculator to solve for  $N$ :  $\log_{10} N = 0.6646$
12. Graph  $y = \log_4 x$  by hand.
13. Determine the domain of  $f(x) = \log_3(x^2 - 9)$ .
14. Graph  $y = \ln(x - 2)$  by hand.
15. True or false:  $\frac{\ln x}{\ln y} = \ln(x - y)$
16. Solve for  $x$ :  $5^x = 41$
17. Solve for  $x$ :  $x - x^2 = \log_5 \frac{1}{25}$
18. Solve for  $x$ :  $\log_2 x + \log_2(x - 3) = 2$
19. Solve for  $x$ :  $\frac{e^x + e^{-x}}{3} = 4$
20. Six thousand dollars is deposited into a fund at an annual percentage rate of 13%.  
Find the time required for the investment to double if the interest is compounded continuously.
21. Use a graphing utility to find the points of intersection of the graphs of  $y = \ln(3x)$  and  $y = e^x - 4$ .
22. Use a graphing utility to find the power model  $y = ax^b$  for the data (1, 1), (2, 5), (3, 8), and (4, 17).